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Energy conversion and neutral surfaces with a nonlinear equation of state

J. Nycander
University of Stockholm, Dept of Meteorology, Stockholm, Sweden (JONAS@MISU.SU.SE, +46 8 164336)

Neutral surfaces are defined so that a water parcel that is displaced adiabatically along such a surface always has the same density as the surrounding water. Since such a displacement does not change the density field or the potential energy, it is generally assumed that it does not produce a restoring buoyancy force. However, it is here shown that, because of the nonlinear character of the equation of state (in particular the thermobaric effect), such a 'neutral´ displacement is accompanied by a conversion between internal and potential energy, and an equal conversion between potential and kinetic energy. While there is thus no net change of potential energy, the kinetic energy does change, implying that there is in fact a restoring force.

It is further shown that displacements that are orthogonal to a vector $\mathbf{P}$ do not induce conversion between potential and kinetic energy, and therefore do not produce a restoring buoyancy force. Hence, the properties usually associated with neutral surfaces, which are orthogonal to the dianeutral vector $\mathbf{N}$, should instead be associated with '$\mathbf{P}$-surfaces', which are orthogonal to $\mathbf{P}$.

To define $\mathbf{P}$, we must first define the specific potential energy $\Pi(\mathbf{r}, t)$. This is the energy required to move a fluid parcel with unit volume from a reference level (e.g. the surface) to its actual depth, taking into account the depth-dependence of the buoyancy force due to the thermobaric effect. Integrating $\Pi(\mathbf{r}, t)$ over the entire fluid volume gives the 'incompressible potential energy' U_B, which is different from the true potential energy U. The sum of the kinetic energy and U_B (but not U) is conserved by the incompressible Boussinesq equations with a nonlinear equation of state in the absence of dissipation.

$\mathbf{P}$ is defined so that a water parcel that is displaced adiabatically along a $\mathbf{P}$-surface always has the same specific potential energy Π as the surrounding water. Such a displacement does not change the Π-field, and therefore also not the incompressible potential energy U_B or the kinetic energy.

The helicity of $\mathbf{P}$ is nonzero, as is the helicity of $\mathbf{N}$. It is therefore impossible to find global surfaces that are everywhere exactly orthogonal to $\mathbf{P}$. Hence, it will be necessary to find approximate $\mathbf{P}$-surfaces by some optimization procedure, similarly as has been done for neutral surfaces.

The vectors $\mathbf{N}$ and $\mathbf{P}$ are not parallel, because of the thermobaric effect. Moreover, the difference between neutral surfaces and $\mathbf{P}$-surfaces is of the same magnitude as the difference between neutral surfaces and surfaces of constant potential density referenced to a widely different depth than the local depth.