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Study and solution of the variational data assimilation problems for the ocean nonlinear hydrothermodynamics model

V. Agoshkov
Institute of Numerical Mathematics RAS, Moscow, Russian Federation (agoshkov@inm.ras.ru, 007 495 9381821)

The development of computational algorithms for the solution of data assimilation problems in geophysical hydrodynamics is one of the important contemporary computation and informational problems [1,2,4]. Some of such problems are studied in this work.

1. Let us consider a spherical coordinate system (λ, θ, r), $z = R - r$, $(\lambda \in [0, 2\pi]$, $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}])$, S_R is the sphere of the radius R, Ω is a part of this sphere, the ocean velocity vector is presented in the coordinate form $U = (u, v, w) \equiv (u, w)$, where $u = (u, v)$ is the 'horizontal' velocity vector and w is the 'vertical' velocity. The sea level will be specified by the equation $z = \xi(\lambda, \theta, t)$, where $(\lambda, \theta, R) \in \Omega$, t is the time variable, $t \in [0, \bar{t}\,]$, $\bar{t} < \infty$, further $f(u) = l + u \sin\theta/(r\cos\theta)$; $n \equiv 1/r$, $m \equiv 1/(r\cos\theta)$, $l = 2\omega\sin\theta$, where ω is the Earth angular velocity.

2. Let us in $D \times (0, \bar{t})$ write the system of the nonlinear hydrothermodynamic equations for the functions u, v, ξ, T, S:

$$\begin{cases} \dfrac{du}{dt} + \begin{bmatrix} 0 & -f \\ f & 0 \end{bmatrix} u - g\,\mathrm{grad}\,\xi + A_u u + (A_k)^2 u \\ \qquad = f - \frac{1}{\rho_0}\,\mathrm{grad}\,P_a - \frac{g}{\rho_0}\,\mathrm{grad}\,\int_0^z \rho_1(T,S)dz' \\ \dfrac{\partial \xi}{\partial t} - m\dfrac{\partial}{\partial x}\Big(\displaystyle\int_0^H \Theta(z) u dz\Big) - m\dfrac{\partial}{\partial y}\Big(\displaystyle\int_0^H \Theta(z)\dfrac{n}{m} v dz\Big) = f_3, \\ \dfrac{dT}{dt} + A_T T = f_T, \quad \dfrac{dS}{dt} + A_S S = f_S, \end{cases} \tag{1}$$

where: $\Theta(z) \equiv r(z)/R$; $f = (f_1, f_2)$, f_3, f_T, f_S are the functions of the 'internal' sources; $g, \rho_0 = const > 0$; T, S are the functions of water temperature and salinity; P_a is a given function.

We consider (1) under some system of boundary conditions (see [2]). Initial conditions are set for the functions u, v, T, S, ξ:

$$u = u^0, v = v^0, T = T^0, S = S^0, \xi = \xi^0 \ \text{ at } \ t = 0, \tag{2}$$

where u^0, v^0, T^0, S^0, ξ^0 are the 'initial data' functions.

One of the important practical case of f in (1) is given by

$$f \equiv g\,\mathrm{grad}\,G \tag{3}$$

for a certain scalar function $G = G(\lambda, \theta, t)$.

In this work we consider the initial - boundary value problem for the system (1) for the 'main unknowns' u, v, ξ, T, S and G, f_3, ξ^0 are considered as the 'additional unknouns'. To complete the problem we use the sea level observation data on the function ξ and apply the variational data assimilation procedure.

3. Suppose the only function obtained by processing observation data is the function ξ_{obs} on $\bar{\Omega} \equiv \Omega \cup \partial\Omega$ at $t \in (0, \bar{t}\,)$. Let the physical meaning of this function be an approximation to the sea level function ξ. A case is admitted when ξ_{obs} exists only in some subset from $\Omega \times (0, \bar{t}\,)$, whose carrier is denoted m_0.

The statement of the problem is the following: find u, v, ξ, T, S, and ξ^0, G, f_3 such that the system (1) with corresponding initial-boundary conditions and the following relation in $\Omega \times (0, \bar{t})$:

$$m_0(\xi - \xi_{obs}) = 0 \quad (4)$$

are satisfied. Then, the relation (4) is reduced to the problem of minimization of a quadratic regularized functional.

To study and solve this problem we introduce the numerical approximations based on splitting methods [3], we apply the optimal control approaches ('variational approaches', 'variational data assimilation procedures', see [1,2,4]) and use the iterative algorithms for solving the minimization problem.

4. Using the above described ocean hydrothermodynamics model complemented by the sea level 'assimilation block' we performed computations for the Indian Ocean with a resolution of $1^\circ \times 0.5^\circ$, in which the assimilation procedure was used to reconstruct G, f_3, ξ^0.

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