



A mixed discontinuous/continuous finite element pair for geophysical flows

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The Imperial College Ocean Model (ICOM) is an advanced three-dimensional, non-hydrostatic adaptive mesh ocean model which includes advances developed in the sphere of industrial fluid mechanics. However, large-scale flows in the atmosphere and ocean remain in a state of slow evolution called geostrophic balance in which fast oscillations are very weak; these slowly-evolving dynamical states represent our weather and the global circulation patterns. To make predictions on these scales, special care must be taken to design numerical methods which reflect the properties of this underlying balance. This calls for new discretisation methods which reflect geostrophic balance. We introduce a new family of mixed finite element methods which combine discontinuous and continuous elements in a new way to produce a stable discretisation which provides the best possible representation of geostrophic balance on unstructured meshes. The proofs of these properties are very simple since they are based on underlying geometric structure: the discretisations have exact sequences which mimic the div-curl and curl-grad relations of vector analysis. We demonstrate that the element exactly preserves geostrophic balance in the shallow water equations and provides an excellent representation of density-driven instabilities even at very low resolution.