A non-linear transfer function time series model consistent with the Data Based Mechanistic framework

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In hydrology a discrete time series of observations $(y_1, \ldots, y_T)$ can often be effectively modelled from an input series $(u_1, \ldots, u_T)$ lagged by τ time steps using a transfer function model of the form

$$y_t + \sum_{i=1}^{n} a_i y_{t-i} = \sum_{j=0}^{m} b_j u_{t-\tau-j} + \eta$$

where the noise η is represented by an ARMA process. In many cases the input series is constructed as a function of multiple observed series e.g. $u_t = y_t^{\phi} r_t$ when constructing an effective rainfall (u_t) from observed rainfall (r_t) and river discharge (y_t). Functions for constructing the input series can often be rewritten as functions for the state dependency of $\mathbf{b} = (b_0, \ldots, b_m)$. Increasingly there is evidence that state dependant formulation for $\mathbf{a} = (a_1, \ldots, a_n)$ many also be required to adequately represent some systems. We outline a method for representing the state dependency of $(\mathbf{a}, \mathbf{b})$ as polynomial functions of observed values. An appropriate robust estimation algorithm using an instrument variable technique is presented. It is shown that the estimation algorithm can be constrained to maintain system stability so allowing interpretation within the Data Based Mechanistic framework of Young (Environmetrics 5, 1994).