



## Semi-analytical solution of three-dimensional transient flow in a coupled $N$ -layer aquifer system

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We present an efficient strategy for solving semi-analytically the transient groundwater head in a coupled  $N$ -layer aquifer system  $\phi_i(r, z, t)$ ,  $i = 1, \dots, N$ , with radial symmetry, with full  $z$ -dependency, and partially penetrating wells. Aquitards are treated as aquifers with their own horizontal and vertical permeabilities. Since the vertical direction is fully taken into account, we do not need to pose the Dupuit assumption, *i.e.* that the flow is mainly horizontal. At the common boundaries of the layers we assume continuity of the head and the flux. At the top and the bottom of the system we assume boundary conditions of Robin type (*i.e.* flow is proportional to the head), including Dirichlet and Neumann conditions.

To solve this problem, we employ the Laplace transform for the  $t$  variable (with transform parameter  $p$ ), the Hankel transform for the  $r$  variable (with transform parameter  $\alpha$ ) and a particular form of a Generalized Fourier transform for the vertical direction  $z$  with an infinite set of eigenvalues  $\lambda_m^2$  (with the discrete index  $m$ ). We solve this problem in the form of a semi-analytical solution by specifying an analytical expression in terms of the variables  $r$  and  $z$ , transform parameter  $p$ , and eigenvalues  $\lambda_m^2(p)$  of the Generalized Fourier transform. The calculation of the eigenvalues  $\lambda_m^2$  and the inversion of these transformed solution to the time domain can only be done numerically.

In this context the application of the Generalized Fourier transform is novel. By means of this Generalized Fourier transform transient problems with horizontal symmetries other than radial can be treated as well.

We demonstrate the capabilities of this technique by an example of particle tracking to and from an partially penetrating well in a system of 6 layers and 3 wells, both under stationary as under transient conditions.