



Probabilistic and geometric descriptions of coherent structures in geophysical flows

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The transport of material constitutes an important aspect of many natural systems. During the last two decades different mathematical concepts have been developed to get a better understanding of the mechanisms of particle transport and to estimate transport rates and probabilities. Areas of application cover many scientific fields, such as fluid dynamics, ocean dynamics, molecular dynamics, physical chemistry, and astrodynamics.

In this contribution we focus on the numerical detection and approximation of coherent structures and barriers to particle transport in flows, connecting the classical geometrical approach via invariant manifolds with a probabilistic approach via transfer operators and almost-invariant sets.

In particular, we analyse several example systems from geophysical applications for different dynamical settings, including the Arnold-Beltrami-Childress (ABC) flow (3d steady non-divergent flow), the Lorenz system (3d dissipative chaotic flow) as well as a double gyre flow (2d periodically forced non-divergent flow). The transfer operator approach directly determines those regions in phase space that minimally mix with their surroundings. The almost-invariant regions are identified via eigenvectors of a transfer operator and are ranked by the corresponding eigenvalues in order of the sets' invariance or "leakiness". While we demonstrate that the corresponding almost-invariant sets are typically bounded by segments of invariant manifolds of hyperbolic objects, without such a ranking it is not at all clear which intersections of invariant manifolds form the major barriers to mixing. Furthermore, we show that boundaries formed by segments of invariant manifolds do not necessarily bound sets of minimal leakage.

Finally, we report on a numerical investigation of three-dimensional circulation in the Southern Ocean. An approximate transfer operator is constructed from velocity data from a state-of-the-art 1/4 degree resolution global ocean model and dominant coherent circulation structures are identified from eigenmodes of this operator.