



## Transitions to Chaos in Dieterich-Ruina Friction

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We began investigations into the Dieterich-Ruina (D-R) friction law in previous work by studying the behavior of a single slider-block under this law. We found transitions to chaos in the numerical solution to this system when a specific parameter was increased. This parameter,  $\epsilon = \frac{B-A}{A}$  is the ratio of the stress parameters ( $B - A$ ) and  $A$  in the D-R friction law. The parameter  $A = \partial\tau/\partial\log(v)$ , where  $\tau$  is the frictional stress and  $v$  is the velocity of the slider, is a measure of the direct velocity dependence (sometimes called the "direct effect") while  $(A - B) = \partial\tau_{ss}/\partial\log(v_{ss})$ , is a measure of the steady-state velocity ( $v_{ss}$ ) dependence. When compared to the slip weakening friction law, the parameter ( $B - A$ ) plays a role of a stress drop while  $A$  corresponds to the strength excess. We found that transitions to chaos for a single block occur for  $\epsilon \approx 9$ . We also studied the behavior of a system of three blocks under the D-R friction law, finding that transitions to chaos occurred for smaller values of this parameter  $\epsilon$ . Taking this study a step further, we derive the elastic wave equation in 1-d under the Dieterich-Ruina friction law to obtain the position  $u(x, t)$ , the slip relative to the driver plate. We solve the resulting PDE by first discretizing in space using the method of lines, and solving the system of ODEs using an explicit 4th order Runge-Kutta numerical scheme. During the spatial discretization, the PDE is reverted to a system of ODEs corresponding to a chain of spring blocks where the spatial mesh determines the number of resulting blocks. Using uniformly distributed initial conditions and periodic boundary conditions, we find that numerical solutions to this PDE bifurcate under critical values of the same parameter  $\epsilon$ . We are interested in values of  $\epsilon$  for which chaotic regimes result, as a function of the spatial mesh, or number of blocks. For 100 blocks, we find that transitions to chaotic solutions occur for  $\epsilon \approx 4.7$ ; a smaller value than the value observed for the chaotic regime for 1 block. Furthermore, our numerical solution of the PDE suggests that for this range of parameter values, the chaotic behavior occurs only in time; the spatial structure of the slip is preserved. We compute the structure function,  $S_1(x, t)$ , to explore statistically stationary states of the solution. We also compute the average of the spectra computed for many runs with different initial conditions in order to study the distribution of frequency. Results from these studies may have implications in the re-normalization of the Dieterich-Ruina friction law.