



## Quasi-geostrophic dynamics of a finite-depth tropopause

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The tropopause plays a crucial role in the dynamics of the atmosphere and simplified models of its balanced dynamics have been obtained using the fact that it corresponds to a sharp transition in stratification. The crudest model consists of describing the tropopause within the quasi-geostrophic approximation as a rigid lid, yielding a surface quasi-geostrophic (SQG) model. The rigid-lid assumption was relaxed by Juckes (1994) who replaced it by the more realistic assumption of a finite jump in the stratification at the tropopause. The resulting model remains the SQG model, with a relationship between potential-temperature and velocity that involves the Brunt-Väisälä frequency  $N$  both of the troposphere and of the stratosphere.

Here we relax the assumption of a vertical jump in stratification at the tropopause and investigate the effect of the internal structure of the tropopause, both in stratification and in shear. Using matched asymptotic expansions, we derive next-order corrections to the SQG model of the tropopause. The model obtained remains three-dimensional, but is focused on the dynamics of the tropopause region, with a simplified inversion relation to obtain the flow outside of that region.

Our reduced model is then applied to study the dynamics of linear perturbations to the tropopause. Edge waves are recovered to leading order, with a correction to the frequency at next order. The sensitivity of this correction to the stratification jump, to the structure of the transition and to the shear are discussed. Now, our model has richer dynamics than the SQG model of the tropopause as it also includes a continuous spectrum of sheared disturbances with no net integral of PV across the tropopause. We show how they coexist with edge waves, how they decay with time because of the shear, and how an arbitrary condition splits into edge waves and sheared disturbances.