



## **Multi-scale analysis of collective behavior in 2D self-propelled particle models of swarms: An Advection-Diffusion with Memory Approach**

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Self-propelled particle models (SPP's) are a class of agent-based simulations that have been successfully used to explore questions related to various flavors of collective motion, including flocking, swarming, and milling. These models typically consist of particle configurations, where each particle moves with constant speed, but changes its orientation in response to local averages of the positions and orientations of its neighbors found within some interaction region. These local averages are based on 'social interactions', which include avoidance of collisions, attraction, and polarization, that are designed to generate configurations that move as a single object. Errors made by the individuals in the estimates of the state of the local configuration are modeled as a random rotation of the updated orientation resulting from the social rules. More recently, SPP's have been introduced in the context of collective decision-making, where the main innovation consists of dividing the population into naïve and 'informed' individuals. Whereas naïve individuals follow the classical collective motion rules, members of the informed sub-population update their orientations according to a weighted average of the social rules and a fixed 'preferred' direction, shared by all the informed individuals. Collective decision-making is then understood in terms of the ability of the informed sub-population to steer the whole group along the preferred direction. Summary statistics of collective decision-making are defined in terms of the stochastic properties of the random walk followed by the centroid of the configuration as the particles move about, in particular the scaling behavior of the mean squared displacement (msd). For the region of parameters where the group remains coherent, we note that there are two characteristic time scales, first there is an anomalous transient shared by both purely naïve and informed configurations, i.e. the scaling exponent lies between 1 and 2. The long-time behavior of the msd of the centroid walk scales linearly with time for naïve groups (diffusion), but shows a sharp transition to quadratic scaling (advection) for informed ones. These observations suggest that the mesoscopic variables of interest are the magnitude of the drift, the diffusion coefficient and the time-scales at which the anomalous and the asymptotic behavior respectively dominate transport, the latter being linked to the time scale at which the group reaches a decision. In order to estimate these summary statistics from the msd, we assumed that the configuration centroid follows an uncoupled Continuous Time Random Walk (CTRW) with smooth jump and waiting time pdf's. The mesoscopic transport equation for this type of random walk corresponds to an Advection-Diffusion Equation with Memory (ADEM). The introduction of the memory, and thus non-Markovian effects, is necessary in order to correctly account for the two time scales present. Although we were not able to calculate the memory directly from the individual-level rules, we show that it can be estimated from a single, relatively short, simulation run using a Mittag-Leffler function as template. With this function it is possible to predict accurately the behavior of the msd, as well as the full pdf for the position of the centroid. The resulting ADEM is self-consistent in the sense that transport parameters estimated from the memory via a Kubo relationship coincide with those estimated from the moments of the jump size pdf of the associated CTRW for a large number of group sizes, proportions of informed individuals, and degrees of bias along the preferred direction. We also discuss the phase diagrams for the transport coefficients estimated from this method, where we notice velocity-precision trade-offs, where precision is a measure of the deviation of realized group orientations with respect to the informed direction. We also note that the time scale to collective decision is invariant with respect to group size, and depends only on the proportion of informed individuals and the strength of the coupling along the informed direction.