



Multivariate singular spectrum analysis and the road to phase synchronization

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Singular spectrum analysis (SSA) and multivariate SSA (M-SSA) are based on the classical work of Kosambi (1943), Loeve (1945) and Karhunen (1946) and are closely related to principal component analysis. They have been introduced into information theory by Bertero, Pike and co-workers (1982, 1984) and into dynamical systems analysis by Broomhead and King (1986a,b). Ghil, Vautard and associates have applied SSA and M-SSA to the temporal and spatio-temporal analysis of short and noisy time series in climate dynamics and other fields in the geosciences since the late 1980s.

M-SSA provides insight into the unknown or partially known dynamics of the underlying system by decomposing the delay-coordinate phase space of a given multivariate time series into a set of data-adaptive orthonormal components. These components can be classified essentially into trends, oscillatory patterns and noise, and allow one to reconstruct a robust "skeleton" of the dynamical system's structure. For an overview we refer to Ghil et al. (Rev. Geophys., 2002).

In this talk, we present M-SSA in the context of synchronization analysis and illustrate its ability to unveil information about the mechanisms behind the adjustment of rhythms in coupled dynamical systems. The focus of the talk is on the special case of phase synchronization between coupled chaotic oscillators (Rosenblum et al., PRL, 1996). Several ways of measuring phase synchronization are in use, and the robust definition of a reasonable phase for each oscillator is critical in each of them.

We illustrate here the advantages of M-SSA in the automatic identification of oscillatory modes and in drawing conclusions about the transition to phase synchronization. Without using any a priori definition of a suitable phase, we show that M-SSA is able to detect phase synchronization in a chain of coupled chaotic oscillators (Osipov et al., PRE, 1996).

Recently, Muller et al. (PRE, 2005) and Allefeld et al. (Intl. J. Bif. Chaos, 2007) have demonstrated the usefulness of principal component analysis in detecting phase synchronization from multivariate time series. The present talk provides a generalization of this idea and presents a robust implementation thereof via M-SSA.