



Finding a suitable finite element for 3D geodynamic modeling

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Numerical modeling of geodynamic processes has become an important research topic in the recent decades. Apart from the correct interpretation of the model results, it is also vastly important to clearly understand the technical aspects of the model formulation, the weaknesses and advantages of different numerical schemes. The majority of numerical models considered to date are two-dimensional. The increased computational complexity of the 3D models make the selection of a suitable numerical scheme even more important compared to 2D.

The geodynamic models can be very ill-posed from the numerical point of view. They usually contain large abrupt viscosity variations, and mixture of compressible and incompressible material behavior. In such a context the spatial discretization becomes very important, because it must provide an accurate and convergent solution, preferably with minimal cost and maximal geometrical flexibility. Practice shows that there is always a trade-off between the accuracy (stability) and the cost. Almost entire 3D finite element codes employed in geodynamic modeling community use some form of spatial discretization that violates the mathematical criteria of stability, the so-called Ladyzhenskaya-Babushka-Brezzi (LBB) condition. The explanation behind this fact is that instabilities do not necessarily show up in practice, but unstable discretization is relatively computationally inexpensive. The stability of LBB-stable elements itself is also conditional and strongly problem-dependend. In the Rayleigh-Taylor benchmark with large (>1000) abrupt viscosity contrasts, discretized with non-fitted mesh, the meaningless velocity solutions can be obtained even with stable elements. Altogether these results bring us to a conclusion that the optimal choice of the finite element still remains unclear and deserves further study.

In this work we bring our recent results concerning the selection of a suitable 3D finite element discretization to the attention of the modeling community. We consider different types of LBB-stable (including quite rear ones) and unstable (stabilized) formulations, both in the context of a standard continuous Galerkin, as well as a discontinuous Galerkin finite element methods. We believe that our findings will be interesting both for the code developers and for the practical researchers in the field of geodynamic modeling.