



## **Geometrical Theory of Spherical Harmonics for Geosciences**

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Spherical harmonics play a central role in the modelling of spatial and temporal processes in the system Earth. The gravity field of the Earth and its temporal variations, sea surface topography, geomagnetic field, ionosphere etc., are just a few examples where spherical harmonics are used to represent processes in the system Earth. We introduce a novel method for the computation and rotation of spherical harmonics, Legendre polynomials and associated Legendre functions without making use of recursive relations. This novel geometrical approach allows calculation of spherical harmonics without any numerical instability up to an arbitrary degree and order, e.g. up to degree and order  $10^6$  and beyond.

The algorithm is based on the trigonometric reduction of Legendre polynomials and the geometric rotation in hyperspace. It is shown that Legendre polynomials can be computed using trigonometric series by pre-computing amplitudes and translation terms for all angular arguments. It is shown that they can be treated as vectors in the Hilbert hyperspace leading to unitary hermitian rotation matrices with geometric properties. Thus, rotation of spherical harmonics about e.g. a polar or an equatorial axis can be represented in the similar way. This novel method allows stable calculation of spherical harmonics up to an arbitrary degree and order, i.e. up to degree and order  $10^6$  and beyond.