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Reproducing Kernels in Harmonic Spaces and Their Numerical Implementation

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In harmonic analysis such as the modelling of the Earth's gravity field, the importance of Hilbert's space of harmonic functions with the reproducing kernel is often discussed. Moreover, in case of an unbounded domain given by the exterior of the sphere or an ellipsoid, the reproducing kernel $K(\mathbf{x},\mathbf{y})$ can be expressed analytically by means of closed formulas or by infinite series. Nevertheless, the straightforward numerical implementation of these formulas leads to dozen of problems, which are mostly connected with the floating-point arithmetic and a number representation. The contribution discusses numerical instabilities in $K(\mathbf{x},\mathbf{y})$ and $\mathbf{grad}\,K(\mathbf{x},\mathbf{y})$ that can be overcome by employing elementary functions, in particular `expm1` and `log1p`. Suggested evaluation scheme for reproducing kernels offers uniform formulas within the whole solution domain as well as superior speed and near-perfect accuracy (10^{-16} for IEC 60559 double-precision numbers) when compared with the straightforward formulas. The formulas can be easily implemented on the majority of computer platforms, especially when C standard library ISO/IEC 9899:1999 is available.