



Conservative adaptivity and two-way self-nesting using discrete wavelets

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In simulating atmosphere and oceans, multiscale modelling is desirable to track high-intensity weather patterns, to investigate the interactions between the various spatio-temporal scales of the climate system, and to perform assessments of climate change at scales small enough to derive impacts on society and ecosystems. The mainstream approach to multiscale modelling is to nest a fine, limited-area model into a coarse, global model. These models are then coupled, either one-way or two-way, in order to combine the global coverage of the global model and the fine details of the fine model.

In the long simulations typical of climate studies, initial conditions are unimportant, except for the few quantities like mass that are exactly conserved. In this context it is crucial that numerical models conserve at least mass exactly at the discrete level. However even with elaborate strategies like adaptive mesh refinement (AMR) conservation is not straightforwardly achieved.

Although the continuous wavelet transform has become a standard tool of geophysical data analysis, it is less known that discrete wavelets and the associated transforms provide the basis for spatially adaptive numerical methods. Such methods are now well-developed in the fluid dynamics community. Since they allow spatial adaptivity, they can also be seen as two-way self-nesting methods. However since they are not specifically designed for geophysical purposes they are usually not exactly conservative.

I present a fairly general framework in which a wavelet-based layer is added to an existing conservative scheme (finite-volume or finite-difference) to make it spatially adaptive without breaking the exact conservation of linear invariants. Discrete wavelet transforms involve an upscaling operation by which fields are transferred from a fine grid to a coarser grid with half the resolution. The method requires that mass fluxes be upscaled in a way that is consistent with the upscaling of mass. This condition is easily met on Cartesian grids, and the method is exemplified with a standard finite-difference discretization of the one-layer rotating shallow water equations.