



## **A conservative method for hydrostatic flow in isentropic coordinates**

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Although our climate is ultimately driven by (nonuniform) solar heating, many aspects of the flow can be understood qualitatively from forcing-free and frictionless dynamics. In the limit of zero forcing and dissipation, our weather system falls under the realm of Hamiltonian fluid dynamics and the flow conserves potential vorticity (PV), energy and phase-space structure.

We have found a conservative numerical scheme for a hydrostatic atmosphere based on a mixed Eulerian-Lagrangian approach, the so-called parcel formulation [1]. For adiabatic flow, the entropy is materially conserved. Under stable stratifications, we introduce isentropic coordinates to simplify the governing equations. The entropic direction is discretized using finite elements. The discretization of horizontal Lagrangian label space (from infinitesimal fluid parcels to discrete fluid particles) yields a discrete Poisson bracket. New is that we apply the Hamiltonian Particle-Mesh method [2], and view the potential as an Eulerian function, reconstructed from the particle data. The use of an Eulerian grid makes the method more efficient and stable. The Hamiltonian consists of a Lagrangian kinetic energy and an Eulerian potential energy. The discrete system of ODE's is thus a Hamiltonian system conserving mass, PV, energy and phase-space structure. If we incorporate a symplectic time integrator, the resulting fully discrete system conserves energy approximately without any drift in energy. Several challenging (nonlinear) solutions will be tested, such a flow over a rising bump. Also, preliminary results for bottom-intersecting isentropes will be demonstrated.

### REFERENCES

- [1] O. Bokhove and M. Oliver, Parcel Eulerian-Lagrangian fluid dynamics for rotating geophysical flows, Proc. Roy. Soc. A. 462, pp. 2563-2573 (2006)
- [2] J. Frank, G. Gottwald, S. Reich, A Hamiltonian particle-mesh method for the rotating shallow-water equations, Lecture Notes in Computational Science and Engineering, Vol. 26, Springer, Heidelberg, pp. 131-142 (2002)