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A Hamiltonian Particle-Mesh Method for Hydrostatic Flow in Isentropic Coordinates

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Although our climate is ultimately driven by (nonuniform) solar heating, many aspects of the flow can be understood qualitatively from forcing-free and frictionless dynamics. In the limit of zero forcing and dissipation, our weather system falls under the realm of Hamiltonian fluid dynamics and the flow conserves potential vorticity (PV), energy and phase-space structure. We think that discrete conservation of phase-space structure is essential for reproducing the correct statistics in climate forecasts. To find a Hamiltonian discretization, we start from the Lagrangian momentum equations with entropy as vertical label coordinate. Entropy is conserved on fluid parcels in adiabatic flow. New is that we view the potential as an Eulerian function. It means that the Lagrangian equation for mass conservation is replaced by an integral relation over all fluid parcels, recovering the Eulerian density field. This is the so-called parcel formulation [1]. The spatial discretization consists of two steps: 1) a finite element (FEM) discretization in the entropic direction, and 2) a discretization of horizontal Lagrangian label space. It leads to a discrete Poisson bracket constituting a Hamiltonian set of ODE's for a collection of discrete fluid particles moving on isentropic surfaces. The resulting Hamiltonian Particle-Mesh method, akin to [3], conserves mass, PV, energy and phase-space structure. A symplectic time integrator is used for our Hamiltonian system. The resulting fully discrete system allows backward error analysis for the energy. It is conserved without any drift in energy. We will prove the energy conservation, and show a systematic derivation of the Eulerian FEM Poisson bracket, following [2]. REFERENCES

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