How might Model-based Probabilities Extracted from Imperfect Models Guide Rational Decisions: The Case for non-probabilistic odds

Leonard A Smith (1,2)

(1) Centre for the Analysis of Time Series, London School of Economics and Political Science, UK (L.smith@lse.ac.uk), (2) Pembroke College, University of Oxford, UK

This contribution concerns "deep" or "second-order" uncertainty, such as the uncertainty in our probability forecasts themselves. It asks the question: "Is it rational to take (or offer) bets using model-based probabilities as if they were objective probabilities?" If not, what alternative approaches for determining odds, perhaps non-probabilistic odds, might prove useful in practice, given the fact we know our models are imperfect? We consider the case where the aim is to provide sustainable odds: not to produce a profit but merely to rationally expect to break even in the long run. In other words, to run a quantified risk of ruin that is relatively small. Thus the cooperative insurance schemes of coastal villages provide a more appropriate parallel than a casino. A "better" probability forecast would lead to lower premiums charged and less volatile fluctuations in the cash reserves of the village. Note that the Bayesian paradigm does not constrain one to interpret model distributions as subjective probabilities, unless one believes the model to be empirically adequate for the task at hand. In geophysics, this is rarely the case.

When a probability forecast is interpreted as the objective probability of an event, the odds on that event can be easily computed as one divided by the probability of the event, and one need not favour taking either side of the wager. (Here we are using "odds-for" not "odds-to", the difference being whether of not the stake is returned; odds of one to one are equivalent to odds of two for one.) The critical question is how to compute sustainable odds based on information from imperfect models. We suggest that this breaks the symmetry between the odds-on an event and the odds-against it.

While a probability distribution can always be translated into odds, interpreting the odds on a set of events might result in "implied-probabilities" that sum to more than one. And/or the set of odds may be incomplete, not covering all events. We ask whether or not probabilities based on imperfect models can be expected to yield probabilistic odds which are sustainable. Evidence is provided that suggest this is not the case. Even with very good models (good in an Root-Mean-Square sense), the risk of ruin of probabilistic odds is significantly higher than might be expected. Methods for constructing model-based non-probabilistic odds which are sustainable are discussed. The aim here is to be relevant to real world decision support, and so unrealistic assumptions of equal knowledge, equal compute power, or equal access to information are to be avoided. Finally, the use of non-probabilistic odds as a method for communicating deep uncertainty (uncertainty in a probability forecast itself) is discussed in the context of other methods, such as stating one's subjective probability that the models will prove inadequate in each particular instance (that is, the Probability of a "Big Surprise").