



Exact and asymptotic local solutions of the one dimensional nonlinear wave equation with degenerate velocity

Brunello Tirozzi (1) and Sergey Dobrokhotov (2)

(1) University of Rome "La Sapienza", Dep. Physics, Rome, Italy (b.tirozzi@libero.it, +39 06 4454749), (2) A.Ishlinsky Institute for Problem of Mechanics, RAS, Prospekt Vernadskogo 101-1, Moscow 119526

In the talk there will be a discussion on the solutions of the Cauchy problem for the equation mentioned in the title, defined in the half line $x > 0$ with variable velocity $c(x) > 0$, such that $c \sim \sqrt{x}$ for small values of x . The initial values have the form $u((x-a)/h)$, where $u(y)$ is a decreasing function for $|y| \rightarrow \infty$, $a > 0$, $h \ll 1$. We discuss and give our interpretation of the well known results obtained by Stoker, Carrier – Greenspan, Pelinovsky-Mazova, also we construct new explicit formulas

for the solutions of the Cauchy problem having the form of isolated waves. The results can be formulated as follows.

1) The Carrier and Greenspan transformation (in practice the hodograph) allows to write the solution of the considered problem

in *parametric form*, expressing it by means of the solution of the linear

wave equation, obtained by a simple linearization of the initial non linear equation.

2) The global (local) solutions of the linear wave equation are found by means of a generalization of the Maslov canonical operator (quasi-classical approximation), so the obtained formula hold also for the solution in the neighborhood of the point $x=0$, which, from the point of view of the geometrical optics, is a focal point. For this reason it is necessary to use the construction of the canonical operator of Maslov, without limiting to the WKB expansion.

3) The change of the form of the wave under reflection from the point $x=0$ and the appearance of the so called N-wave (using the terminology of Pelinovsky and Mazova is connected with the increase of the Maslov index when there is the reflection from this point.

4) For some smart choice of the initial condition $u((x-a)/h)$, obtained from the analysis of the two-dimensional wave equation, and in the case of the velocity $c = \alpha\sqrt{x}$, $\alpha = \text{const}$, we obtain *exact explicit formula* for the solution (also in the neighborhood of the point $x=0$), which can be defined by means of the roots of some rational function.

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