



## General data assimilation scheme based on diffusion-type process approximation for linear or non-linear models

konstantin belyaev, giovanni ruggiero, and Clemente A.S. Tanajura

Federal University of Bahia, Salvador, Brazil; Shirshov Institute of Oceanology, Moscow, Russia

A data assimilation method based on a special type of stochastic process approximation is considered and applied. Unlike the standard Kalman-filter theory, this method does not require model linearity and can be rigorously proved for arbitrary non-linear model under some reasonable conditions. In practice, it can be coupled with an ocean circulation model used as a black-box. The idea of the method is to use the diffusion process and, hence, the corresponding Fokker-Planck equation as an evolution law for the covariance function of the model error. In order to avoid the complex numerical computations that appear in the numerical solution of Fokker-Planck equations, further simplifications can be done. In particular, the perturbation theory allows simplifying the form of Fokker-Planck equation and reducing the scheme to ordinary differential equations.

The Fokker-Planck equation for the joint probability of the temperature-salinity error is written as:

$$\frac{\partial p(t, x)}{\partial t} = -\frac{\partial(Ap)}{\partial x} + \frac{1}{2} \frac{\partial^2(Bp)}{\partial x^2(1)}$$

Here,  $p$  is the probability of the error,  $t$  is time,  $x$  is the value of the error, i.e. the difference between model and observation (temperature and/or salinity). Coefficients  $A$  and  $B$ , the drift vector and diffusion matrix, respectively, are defined from the ensemble experiments as described in [1]. Equation (1) can be expanded with respect to a small parameter  $\kappa = \frac{a_0}{\sigma_0}$ , where  $a_0 = \int \|A(t, x)\| dx$  and  $\sigma_0 = \int \sqrt{\|B\|} dx$ . The first two orders of this expansion lead to

$$\frac{\partial p_0(t, x)}{\partial t} = \frac{1}{2} \frac{\partial^2(b^{2p_0})}{\partial x^2(2)}$$

$$\frac{\partial p_1(t, x)}{\partial t} = -\frac{\partial(Ap_0)}{\partial x} + \frac{1}{2} \frac{\partial^2(b^{2p_1})}{\partial x^2(3)}$$

These two equations can be solved sequentially and analytically with given initial conditions. After determining the solution of (2), the joint probability is used to find out the sought covariance and the assimilation problem is completed.

This method has been applied to the HYCOM (Hybrid Coordinate Ocean Model) [2] and ARGO and PIRATA daily data set for the Atlantic Ocean over the period January-March 2008. There were used around 200 temperature and salinity profiles per month. The simulation and assimilation results have been compared with independent data assimilation experiments made with other techniques, in particular with local ensemble Kalman filter scheme and the optimal interpolation method [3].

The results of this work lead to the conclusion that the proposed method is competitive, feasible, mathematically rigorous and numerically effective. It can be applied for operational assimilation and its skills is comparable with existing assimilation methods, for instance, with the Kalman-filter scheme.

### References

1. Belyaev, K., Tanajura, C.A.S., O'Brien J.J., A data assimilation technique with an ocean circulation model and its application to the tropical Atlantic, Appl. Math. Model. 25 (2001) 655–670.

2. Bleck, R. An oceanic general circulation model framed in hybrid isopycnic-cartesian coordinates. *Ocean Modelling*, 4 (2002), 55-88.
3. Kalnay, E., *Atmospheric Modeling, Data Assimilation and Predictability*. Cambridge University Press, (2003) 341 pp.