



Mathematical modelling of plant water and nutrient uptake

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In this presentation I will describe a model of plant water and nutrient uptake and how to translate this model and experimental data from the single root scale to the root branching structure scale. The model starts at the single root scale and describes the water and nutrient movement in the soil using Richards' equation (water uptake) and diffusion-convection equation (nutrient uptake). The water and nutrient uptake in the single root scale model is represented by boundary conditions. In the case of nutrient uptake this has the form of a non-linear Michaelis-Menten uptake law and in the case of water this is given by a soil-xylem pressure difference boundary condition. The flow of water in the xylem is modeled as Poiseuille flow. We solve the single root scale models using the analytic approximate technique of asymptotic expansions similar to Oseen expansions known from fluid dynamics.

We will then discuss how to use the analytic expression to estimate the water and nutrient uptake by growing root branching systems. We model the growth of the root system using a dynamic population model to describe the branching and elongation of roots in the branching system. This root branching population model results in a hyperbolic equation similar to age dependent population models and it can be solved fully analytically using the method of characteristics. Thus we have a fully analytic description of the root branching system evolution. We use this branching model to estimate the nutrient uptake in a scenario when the competition between subbranches is small, i.e., as it is in the case of phosphate, potassium and arsenic. We compare our approximate analytic model to a full 3d simulation of the root system phosphate uptake and find that the analytic model almost perfectly reproduces the 3d numerical model. In addition the analytic model can be included in larger field/catchment/climate scale models something which is not practically possible with the numerical simulations due to their high computational burden.

As a further development of the analytic model we extend it to take into account more details about the root morphology, such as the branching angle between roots, to calculate the evolution of the soil moisture and nutrient concentration profiles due to surface fertilisation and rainfall events. Using this model we are able to determine the relationship between the rainfall events and fertiliser movement into the soil profile. We find that there is a critical rate of rainfall below which the fertilizer (or pollutant) movement into the deeper layers of the soil is impeded due to the development of a slowly varying fluid saturation profile.