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Gravity Waves, Scale Asymptotics and the Pseudo-Incompressible Equations

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The Boussinesq equations provide a convenient modeling framework for studies of gravity-wave dynamics not affected by the impact of varying density on the wave amplitude. In the atmosphere, however, gravity waves undergo tremendous amplitude growth in their upward propagation since atmospheric density has a very strong vertical dependence. This effect, leading to all kinds of wave instabilities, is decisive for the corresponding wave mean-flow interaction. A study of these processes within the complete Euler equations is complicated by their incorporation of sound waves which might at most be of secondary importance in this context. A way around this is the use of an approximated equation set, filtered of sound waves, but representing the dynamics of gravity waves at good accuracy. Both the classic anelastic equations and the pseudo-incompressible equations offer themselves for this purpose. The question arises which of the two, if any, are consistent with a rigorous multiple-scale asymptotics of gravity-wave dynamics in the atmosphere. Hence, such an asymptotics is used to analyze the Euler equations so that the dynamical situation of a gravity wave (GW) near breaking level is best approximated. A simple saturation argument is used to obtain a potential-temperature wave scale, while linear theory yields from the latter the velocity scale, and the wave Exner pressure scale. It also determines the time scale once the spatial scale has been set. As small expansion parameter the product of vertical wave number and potential temperature scale height is used. It is shown that the resulting equation hierarchy is consistent with that obtained from the pseudo-incompressible equations, both for non-hydrostatic and hydrostatic gravity waves. This gives a mathematical justification for the use of the pseudo-incompressible equations for studies of gravity-wave breaking in the atmosphere. An analogous argument does not seem to exist for the anelastic equations