



Full wave description of VLF wave penetration through the ionosphere

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Of the many problems in whistler study, wave propagation through the ionosphere is among the most important, and the most difficult at the same time. Both satellite and ground-based investigations of VLF waves include considerations of this problem, and it has been in the focus of research since the beginning of whistler study (Budden [1985]; Helliwell [1965]). The difficulty in considering VLF wave passage through the ionosphere is, after all, due to fast variation of the lower ionosphere parameters as compared to typical VLF wave number. This makes irrelevant the consideration in the framework of geometrical optics, which, along with a smooth variations of parameters, is always based on a particular dispersion relation. Although the full wave analysis in the framework of cold plasma approximation does not require slow variations of plasma parameters, and does not assume any particular wave mode, the fact that the wave of a given frequency belongs to different modes in various regions makes numerical solution of the field equations not simple. More specifically, as is well known (e.g. Ginzburg and Rukhadze [1972]), in a cold magnetized plasma, there are, in general, two wave modes related to a given frequency. Both modes, however, do not necessarily correspond to propagating waves. In particular, in the frequency range related to whistler waves, the other mode is evanescent, i.e. it has a negative value of N^2 (the refractive index squared). It means that one of solutions of the relevant differential equations is exponentially growing, which makes a straightforward numerical approach to these equations despairing. This well known difficulty in the problem under discussion is usually identified as numerical swamping (Budden [1985]). Resolving the problem of numerical swamping becomes, in fact, a key point in numerical study of wave passage through the ionosphere. As it is typical of work based on numerical simulations, its essential part remains virtually hidden. Then, every researcher, in order to get quantitative characteristics of the process, such as transmission and reflection coefficients, needs to go through the whole problem. That is why the number of publications dealing with VLF wave transmission through the ionosphere does not run short.

In this work, we develop a new approach to the problem, such that its intrinsic difficulty is resolved analytically, while numerical calculations are reduced to stable equations solvable with the help of a routine program. Using this approach, the field of VLF wave incident on the ionosphere from above is calculated as a function of height, and reflection coefficients for different frequencies and angles of incidence are obtained. In particular, for small angles of incidence, for which incident waves reach the ground, the reflection coefficient appears to be an oscillating function of frequency.

Another goal of the work is to present all equations and related formulae in an undisguised form, in order that the problem may be solved in a straightforward way, once the ionospheric plasma parameters are given.

References

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