



Lagrangian Coherent Structures, Hyperbolicity, and Lyapunov Exponents

George Haller

Department of Mechanical Engineering, McGill University, Montreal, Canada

We review the fundamental physical motivation behind the definition of Lagrangian Coherent Structures (LCS) and show how it leads to the concept of finite-time hyperbolicity in non-autonomous dynamical systems. Using this concept of hyperbolicity, we obtain a self-consistent criterion for the existence of attracting and repelling material surfaces in unsteady fluid flows, such as those in the atmosphere and the ocean.

The existence of LCS is often postulated in terms of features of the Finite-Time Lyapunov Exponent (FTLE) field associated with the system. As simple examples show, however, the FTLE field does not necessarily highlight LCS, or may highlight structures that are not LCS. Under appropriate nondegeneracy conditions, we show that ridges of the FTLE field indeed coincide with LCS in volume-preserving flows. For general flows, we obtain a more general scalar field whose ridges correspond to LCS.

We finally review recent applications of LCS techniques to flight safety analysis at Hong Kong International Airport.