



Canonical Hamiltonian representation of pseudoenergy in shear flows using counter-propagating Rossby waves

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Pseudoenergy serves as a non-canonical Eulerian Hamiltonian of linearized shear flow systems. It is non-canonical in the sense that the canonical Hamilton equations cannot be written when the dynamical variable is taken as the (potential) vorticity. Here we apply the counter-propagating Rossby wave kernel (KRW) perspective to obtain a compact form of the pseudoenergy as a domain integral of the local KRW pseudomomentum carried by the instantaneous KRW phase speed in the mean flow frame of reference. Written this way, with the generalized momenta taken as the KRW pseudomomenta and the generalized coordinates as the instantaneous KRW locations, canonical Hamilton equations can be derived both in their continuous (using functional derivatives) and discrete (using function derivatives) forms.

As a simple example of the insight such a formulation can yield, we reexamine the classical stability transition from Rayleigh to Couette flow. In this transition the instability is lost even though the classical necessary conditions of Fjørtoft and Rayleigh are still satisfied. The pseudoenergy-KRW formulation allows to interpret the stabilization both as an inability of the KRWs to phase lock constructively, and in terms of the pseudoenergy becoming negative. These two apparent different rationalizations are shown to be essentially one and the same.