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Extended modified Korteweg – de Vries equation for internal gravity waves in a symmetric three-layer fluid

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Three-layer stratification is proved to be a proper approximation of sea water density and background current profiles in some basins in the World Ocean with specific hydrological conditions. Such a medium is interesting from the point of view of internal gravity wave dynamics, because, in the symmetric about mid-depth case (equal thicknesses of the lower and the upper layers, equal small density jumps on the interfaces), it leads to disappearing of quadratic nonlinearity when described in the framework of weakly nonlinear evolutionary models, which are derived through the asymptotic expansion in small parameters of nonlinearity and dispersion. In this situation the nonlinear transformation of the internal wave disturbances, as is customary, is determined by the influence of the next-order – cubic – nonlinear term in asymptotic series, and for three-layer fluid model the cubic nonlinearity coefficient can have either sign depending on the layer depths (in contrast to traditional two-layer approximation, for which cubic nonlinearity is always negative). Appropriate nonlinear evolutionary equation is modified Korteweg – de Vries equation (mKdV). It is well-known integrable equation of KdV-type, providing solitary wave and breather solutions for positive cubic nonlinearity. The property of sign change for cubic nonlinear coefficient in the mKdV for internal gravity waves in symmetric three-layer fluid requires taking into account next-order nonlinear term (or terms), therefore higher-order extensions of mKdV equation are necessary to provide improved description of internal wave processes.

In the present study we derive nonlinear evolution equations for both interfaces in symmetric three-layer fluid (under Boussinesq approximation) up to the fourth order in small parameters of nonlinearity (ϵ) and dispersion (μ). Applying mKdV-scaling for ratio of these parameters ($\mu = \epsilon^2$) we obtain high-order mKdV equations for interfaces (they have different signs of even-power nonlinear and nonlinear dispersion terms). These equations include additional terms of nonlinearity (fourh and fifth power), nonlinear dispersion and linear dispersion (fifth derivative). Coefficients of these equations are found in the explicit form as functions of medium parameters (layer depths, densities), their signs are analyzed. But the equations derived are too complex for the analysis of the nonlinear wave dynamics, therefore a simplifying asymptotic nonlinear transformation of wavefield is suggested, which reduces these equations to a simpler equation having a form of mKdV equation with additional fourth and fifth power nonlinear terms. But for the particular case of three-layer symmetric fluid fourth power nonlinear correction has zero coefficient, and final equation has only one additional term: fifth power nonlinearity. Its coefficient after the transformation is negative for symmetric three-layer stratification. It is worth to notice that equations for both interfaces are reduced to the same equation, but the asymptotic transformations for the displacements of the interfaces differ by the sign of one term proportional to the square of the displacement amplitude. Thus, the asymptotic transformation introduces an asymmetry of the interfacial displacements in a clear, explicit form in contrast to complex high-order equations.

Next, we considered the equation obtained after transformation (we call it "extended mKdV"), and found its one-soliton solutions for positive cubic nonlinearity. These solitary waves can have either polarity, as well as solitons of mKdV equation, but they have amplitude limit, and while their amplitude grows up to this limit, solitons become wider in much the same manner as the solitons of Gardner equation (extended KdV equation with quadratic and cubic nonlinear terms) in a case of negative cubic nonlinearity (corresponding to internal waves in two-layer fluid).

The solitary wave solutions of the improved weakly nonlinear theory are compared to the fully nonlinear solitary-

like waves numerically simulated in the framework of Euler equations for slightly smoothed symmetric three-layer fluid with small density jumps. Qualitatively both solutions are in good agreement, but quantitative estimates show that improved weakly nonlinear theory underestimates limiting amplitude of solitary waves for about 30%.