Energetics of tidally generated internal waves for nonuniform stratification

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In theoretical work on the generation of internal waves from tides, the WKB-approximation is often used when considering nonuniform stratification. As a result, the rate of energy conversion to internal waves is proportional to the buoyancy frequency at the bottom, \( N_B \). The WKB-approximation requires that the vertical wave length of the internal waves is shorter than the characteristic length scale for variations of the buoyancy frequency \( N(z) \). This is true for higher vertical wave modes, but not for the low modes. For example, the wave length of the lowest mode is of the same order as the entire ocean depth.

In order to understand the effect of nonuniform stratification on the generation of the lowest modes, a homogenization technique is here used. This approach is based on the assumption that the vertical wave length of the internal waves is longer than the characteristic length scale for variations of \( N(z) \), i.e. it is valid in the opposite regime as the WKB-approximation. \( N(z) \) is then replaced by the homogenized profile \( N^h(z) \), which is obtained by averaging \( N^2 \) over the homogenization scale. The result is that the energy conversion is proportional to \( N^h_B \), i.e. to \( N(z) \) homogenized approximately over a vertical wavelength from the bottom. For the lowest mode, \( N^2(z) \) is thus averaged over the entire ocean depth. With a realistic stratification, \( N^h_B \) is much larger than \( N_B \). The WKB-approximation therefore severely underestimates the energy conversion for the lowest modes.

The energy conversion at a simple ridge is calculated for different values of the ridge width, using a realistic profile of \( N(z) \). The results are obtained both numerically (the exact result), and using the WKB-approximation and the homogenization technique, respectively. When the WKB-approximation is used (or when the stratification is uniform), the energy conversion is largest in the limit of zero width, as has been noted previously. However, the numerical result shows that the conversion in fact has a maximum for an intermediate width, of the order 10 km. The reason is that a wider ridge projects more strongly onto the lowest modes, and the conversion into these modes depend on the value of \( N(z) \) higher up in the water column, where it is larger than near the bottom. This effect is missed entirely by the WKB-approximation. For a still wider ridge, the conversion again decreases, since it becomes too wide to project strongly onto even the lowest mode.