



Three-dimensional radiative instabilities in stratified compressible flows

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We investigate the three-dimensional stability of two-dimensional flows $U_x(z)$ such as plane Bickley jet or boundary layer in a stably stratified fluid of constant Brunt-Väisälä frequency N . The angle θ between the shear x - z plane and the vertical is considered as a control parameter. For $\theta = 0$, the gravitational acceleration is along the $(-z)$ -axis so the shear plane is vertical and for $\theta = \pi/2$, along the y -axis thus orthogonal to the stratification. Following the parallel flow approximation, the instability wave solution is sought in the form of a normal mode in the x and y directions: $p \sim \tilde{p} \exp(i\omega t - ik_x x - ik_y y)$. The eigenvalue problem for ω is solved with a spectral collocation method. For radiative instability waves, an appropriate contour deformation in the complex z -plane is developed in order to apply correctly the condition of radiation in numerical codes.

It emerges that, as a wave propagates downstream, its transverse behaviour may be dispersive. This phenomenon depends strongly on the stratification of the mean flow, through the parameter θ . For $\theta = 0$, Miles (1961) and Howard (1961) found a sufficient condition for stability, in terms of Richardson number. The case associated with $\theta = \pi/2$ for a Bickley jet was studied recently by Deloncle *et al* (2007) under Boussinesq approximation. But the way such results may be extended into a wider context, for a given θ , cannot be simply deduced from their analysis. We draw attention to a mechanism whereby gravity waves may be generated by the mean flow without necessarily invoking Kelvin-Helmholtz instabilities. The radiating modes found in our study are similar to the “unstable gravity waves” mentioned by Satomura (1980) in shallow water or “supersonic instability waves” described by Tam and Hu (1988,1989) for high-speed jets. From a mathematical point of view, it can be established that the modes with a real part of frequency satisfying $k_x N / \sqrt{k_x^2 + k_y^2} < \omega_r < N$, where N is the Brunt-Väisälä frequency, give rise to gravity waves in the transverse direction.

The boundary layer flow is studied under fully compressible framework. The radiative instability still occurs and is analysed by using different asymptotic approaches, for large wavenumbers k_x and k_y or for small Brunt-Väisälä frequency. In both cases, the most unstable modes properties are captured in those limits.