



The dynamics of a low-order model for the Atlantic Multidecadal Oscillation

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Observations and model studies provide ample evidence for the presence of multidecadal variability in the North Atlantic sea-surface temperature known as the Atlantic Multidecadal Oscillation (AMO). This variability is characterised by a multidecadal time scale, the westward propagation of temperature anomalies, and a phase difference between the anomalous meridional and zonal overturning circulations.

We derive a low-order model which captures the characteristics of the AMO. The starting point is a minimal model consisting of a thermal wind balance and an equation for the advection of temperature in a 3-dimensional box-shaped ocean basin. The low-order model is obtained by an orthogonal projection onto a finite-dimensional function space. Flows are forced by restoring the sea surface temperature to an idealised atmospheric temperature profile with an equator-to-pole gradient ΔT as a control parameter. A second control parameter, γ , interpolates between restoring ($\gamma = 0$) and prescribed heat flux ($\gamma = 1$) conditions.

For the standard values $\Delta T = 20^\circ\text{C}$ and $\gamma = 0$ the low-order model has a stable equilibrium which corresponds to a steady ocean flow. By increasing the parameter γ from 0 to 1 this equilibrium becomes unstable through a supercritical Hopf bifurcation and we find a periodic attractor with the physical signature of the AMO. In turn, this attractor can bifurcate through (cascades of) period doublings when ΔT is increased. Next, we impose a time-periodic forcing, modelling annual variations in the ocean-atmosphere heat flux. In this setting the AMO appears through a Hopf-Neimark-Sacker bifurcation as an invariant 2-torus attractor. Hence, we have to study bifurcations of invariant tori. For $\Delta T \geq 22^\circ\text{C}$ the 2-torus associated with the AMO bifurcates through a sequence of quasi-periodic period doublings, which can give birth to strange attractors of *quasi-periodic Hénon-like* type.