



## Convolution and non convolution Perfectly Matched Layer techniques optimized at grazing incidence for high-order wave propagation modelling

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We present and discuss here two different unsplit formulations of the frequency shift PML based on convolution or non convolution integrations of auxiliary memory variables. Indeed, the Perfectly Matched Layer absorbing boundary condition has proven to be very efficient from a numerical point of view for the elastic wave equation to absorb both body waves with non-grazing incidence and surface waves. However, at grazing incidence the classical discrete Perfectly Matched Layer method suffers from large spurious reflections that make it less efficient for instance in the case of very thin mesh slices, in the case of sources located very close to the edge of the mesh, and/or in the case of receivers located at very large offset. In [1] we improve the Perfectly Matched Layer at grazing incidence for the seismic wave equation based on an unsplit convolution technique. This improved PML has a cost that is similar in terms of memory storage to that of the classical PML. We illustrate the efficiency of this improved Convolutional Perfectly Matched Layer based on numerical benchmarks using a staggered finite-difference method on a very thin mesh slice for an isotropic material and show that results are significantly improved compared with the classical Perfectly Matched Layer technique. We also show that, as the classical model, the technique is intrinsically unstable in the case of some anisotropic materials. In this case, retaining an idea of [2], this has been stabilized by adding correction terms adequately along any coordinate axis [3]. More specifically this has been applied to the spectral-element method based on a hybrid first/second order time integration scheme in which the Newmark time marching scheme allows us to match perfectly at the base of the absorbing layer a velocity-stress formulation in the PML and a second order displacement formulation in the inner computational domain. Our CPML unsplit formulation has the advantage to reduce the memory storage of CPML by 40% in 2D comparing to the GFPML split formulation of [4]. Examples of waves propagating in heterogeneous thin slices in presence of free surface are shown.

We also applied this CPML technique to more complex models like poroelastic [5] or viscoelastic [6] media based on a fourth-order staggered finite-difference method. For the two-dimensional Biot poroelastic equations we show its efficiency for both non dissipative and dissipative Biot porous models. For the three-dimensional viscoelastic seismic wave equation, the time marching equations of the standard linear solid mechanisms used do not need to be split and only the memory variables associated with velocity derivatives are stored at each time step. In the case of more than one damping mechanism, we are able to reduce memory storage by more than 70% in the PML regions in 3D simulations compared to split PMLs optimized at grazing incidence. Benchmarks of the CPML technique have been validated in poroelastic or viscoelastic thin mesh slices.

These unsplit CPMLs are usually computed based on a second-order finite-difference time scheme. However, in many situations like very long time simulations, it is of interest to increase the accuracy of the method by increasing the order of the time marching and spatial discretizations. The CPML is not able to be increased at high orders because of its convolution formulation. In [7] we study then how to build a new unsplit PML (ADE-PML/Auxiliary Differential Equations PML) that remains optimized at grazing incidence based on a high-order time scheme like the fourth-order Runge-Kutta scheme. At second order in time we demonstrate that

CPML and ADE-PML are equivalent. At second or high order discretization in time, explicit and semi-implicit solutions can be obtained with very good accuracy.

## References

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