



4DVar Assimilation in the Unstable Subspace : existence of an optimal subspace dimension

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The nonlinear stability properties of a chaotic system are exploited to formulate a reduced subspace 4-dimensional assimilation algorithm, 4DVar-AUS (Assimilation in the Unstable Subspace). The key result is the existence of an optimal subspace dimension for the assimilation that is directly related to the unstable subspace dimension. Theoretical arguments suggest that the optimal subspace dimension is equal to $N+1$, where $N+$ is the number of nonnegative Lyapunov exponents. In support of the theory, numerical experiments are performed in a simple model with a variable number of positive exponents: the results show that, in the presence of observational error, the confinement of the assimilation increment in the unstable subspace of the system reduces the RMS analysis error with respect to standard 4DVar. The standard 4DVar solution, while being closer to the observations, is further away from the truth. The explanation of this result is that, assimilating in the unstable subspace, errors in the stable directions are naturally damped: because of observational error, assimilating the whole space otherwise prevents this decay. In agreement with this interpretation, if observations are perfect standard 4DVar gives the best results.

These results are in agreement with an independent theoretical study of the Extended Kalman Filter, which show that the error concentrates in the unstable subspace.