Development and testing of a simple inertial formulation of the shallow water equations for flood inundation modelling

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This abstract describes the development of a new set of equations derived from 1D shallow water theory for use in 2D storage cell inundation models. The new equation set is designed to be solved explicitly at very low computational cost, and is here tested against a suite of four analytical and numerical test cases of increasing complexity. In each case the predicted water depths compare favourably to analytical solutions or to benchmark results from the optimally stable diffusive storage cell code of Hunter et al. (2005). For the most complex test involving the fine spatial resolution simulation of flow in a topographically complex urban area the Root Mean Squared Difference between the new formulation and the model of Hunter et al. is ~1 cm. However, unlike diffusive storage cell codes where the stable time step scales with \((1/\Delta x)^2\) the new equation set developed here represents shallow water wave propagation and so the stability is controlled by the Courant–Freidrichs–Lewy condition such that the stable time step instead scales with \(1/\Delta x\). This allows use of a stable time step that is 1-3 orders of magnitude greater for typical cell sizes than that possible with diffusive storage cell models and results in commensurate reductions in model run times. The maximum speed up achieved over a diffusive storage cell model was 1120x in these tests, although the actual value seen will depend on model resolution and water depth and surface gradient. Solutions using the new equation set are shown to be relatively grid-independent for the conditions considered given the numerical diffusion likely at coarse model resolution. In addition, the inertial formulation appears to have an intuitively correct sensitivity to friction, however small instabilities and increased errors on predicted depth were noted when Manning’s \(n = 0.01\). These small instabilities are likely to be a result of the numerical scheme employed, whereby friction is acting to stabilise the solution although this scheme is still widely used in practice. The new equations are likely to find widespread application in many types of flood inundation modelling and should provide a useful additional tool, alongside more established model formulations, for a variety of flood risk management studies.