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Solar tidal variations of coefficients of second harmonic of gravitational potential of Mercury

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Abstract. Variations of coefficients of the second harmonic of Mercury potential caused by the solar tides have been studied. In the paper we use analytical expressions for tidal variations of Stoks coefficients obtained for model of the elastic celestial body with concentric distributions of masses and elastic parameters (Love numbers) and their reduced form with using fundamental elastic parameter k_2 of the Mercury. Taking into account the resonant properties of the Mercury motion variations of the Mercury potential coefficients we present in the form of Fourier series on the multiple of corresponding arguments of the Mercury orbital theory. Evaluations of the amplitudes and periods of observed variations of Mercury potential have been tabulated for base elastic model of the Mercury characterized by hypothetic elastic parameter (Love number) k_2=0.37 (Dehant et al., 2005). Tidal variations of polar moment of inertia of the Mercury (due to tidal deformations) lead to remarkable variations of the Mercury rotation. Tidal variations of the Mercury axial rotation also have been determined and tabulated. From our results it follows that the tide periodic variations of gravitational coefficients of the Mercury in a few orders bigger then corresponding tidal variations of Earth's geopotential coefficients (Ferrandiz, Getino, 1993).

Variations coefficients of the second harmonic of Mercury potential. These variations are determined by the known formulae for variations of coefficients of the second harmonic of geopotential (Ferrandiz, Getino, 1993). Here we present these formulae in some special form as applied to the considered problem about the Mercury tidal deformations:

$$\delta J_2 = -3T\alpha_3^2/2, \quad \delta C_{22} = T\left(\alpha_1^2 - \alpha_2^2\right)/4, \quad \delta S_{22} = T\alpha_1\alpha_2/2, \quad \delta C_{21} = T\alpha_1\alpha_3, \quad \delta S_{21} = T\alpha_2\alpha_3.$$

Here $T = k_2\left(MR^3/ma^3\right) = 1.667 \cdot 10^{-7}$ is a estimation of some conditional coefficient of tidal deformation of Mercury. m and $R$are the mass and the mean radius of Mercury. Here we have used standard values of ratio of mass of the Sun and Mercury $m/M =$ 6023600, mean radius of Mercury $R =$ 2439.7 km. a = 0.3870983098 AU is an unperturbed value of major semi-axis of Mercury orbit. k_2=0.37. $\alpha_j$is direction cosines of the radius-vector of the Sun in Mercury principal axes of inertia.

The central problem of the work was a construction of trigonometric developments of the producta and squares of these direction cosines multiplied on function$(a/r)^3$, where r is a value of radius-vector of the Sun and$a$ is a major semi-axis of orbit of Mercury (unperturbed value): $(a/r)^3\,\alpha_i\alpha_j$. Omiting sufficiently long procedure on construction of developments for mentioned products we present final formulas for solar tidal variations of coefficients of Mercury gravitational potential:

$$\delta J_2 = -3k_2\frac{M}{m}\left(\frac{R}{a}\right)^3 \sum_{\nu}\left[R_{0,\nu}(\rho,t)\cos\Theta_\nu + r_{0,\nu}(\rho,t)\sin\Theta_\nu\right]$$

$$\delta S_{22} = -\frac{1}{8}k_2\frac{M}{m}\left(\frac{R}{a}\right)^3 \sum_{\nu}\sum_{\varepsilon}\left[R_{2,\nu}^{(\varepsilon)}\cos\left(2g+2l-\varepsilon\Theta_\nu\right) - r_{2,\nu}^{(\varepsilon)}\sin\left(2g-\varepsilon\Theta_\nu\right)\right],$$

$$\delta C_{21} = -\frac{1}{4}k_2\frac{M}{m}\left(\frac{R}{a}\right)^3 \sum_{\nu}\sum_{\varepsilon}\left[R_{1,\nu}^{(\varepsilon)}\cos\left(g+l-\varepsilon\Theta_\nu\right) - r_{1,\nu}^{(\varepsilon)}\sin\left(g+l-\varepsilon\Theta_\nu\right)\right],$$

$$\delta S_{21} = -\frac{1}{4}k_2\frac{M}{m}\left(\frac{R}{a}\right)^3\sum_{\nu}\sum_{\varepsilon}\left[R^{(\varepsilon)}_{1,\nu}\cos\left(g+l-\varepsilon\Theta_\nu\right) - r^{(\varepsilon)}_{1,\nu}\sin\left(g-\varepsilon\Theta_\nu\right)\right].$$

For simplicity here we put the value of the angle $\theta = 0^0$, that means that in unperturbed rotational motion of Mercury its vector of angular momentum consides with the polar principial axis of inertia. Here $\varepsilon = \pm 1$; Θ_ν are arguments located on multiple of mean longitudes of planets (Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uran and the Neptune): $\Theta_\nu = \nu_1 L_{Me} + \nu_2 L_V + \nu_3 L_E + \nu_4 L_{Ma} + \nu_5 L_{Ju} + \nu_6 L_{Sa} + \nu_7 L_{Ur} + \nu_8 L_{Ne}$; $\nu = (\nu_1, \nu_2, \nu_3, ..., \nu_8)$ are corresponding sets of integer indexes. Here all functions R and r are special inclination functions depending from angle $\rho$of inclination of vector of angular momentum of Mercury with respect to normal to base (Laplace) plane and coefficients:$A^{(j)}_\nu$, $B^{(j)}_\nu$ and $a^{(j)}_\nu$, $b^{(j)}_\nu$:

$$R_{0,\nu}(\rho,t) = -\frac{1}{6}\left(3\cos^2\rho - 1\right)A^{(0)}_\nu - \frac{1}{2}\sin 2\rho A^{(1)}_\nu - \frac{1}{4}\sin^2\rho A^{(2)}_\nu,$$

$$r_{0,\nu}(\rho,t) = -\frac{1}{6}\left(3\cos^2\rho - 1\right)a^{(0)}_\nu - \frac{1}{2}\sin 2\rho a^{(1)}_\nu - \frac{1}{4}\sin^2\rho a^{(2)}_\nu,$$

$$R^{(\varepsilon)}_{1,\nu} = \sin 2\rho\left(A^{(0)}_\nu - \frac{1}{2}A^{(2)}_\nu\right) - 2\cos 2\rho A^{(1)}_\nu + 2\varepsilon\cos\rho B^{(1)}_\nu - \varepsilon\sin\rho B^{(2)}_\nu,$$

$$r^{(\varepsilon)}_{1,\nu} = 2\cos\rho b^{(1)}_\nu - \sin\rho b^{(2)}_\nu - \varepsilon\sin 2\rho\left(a^{(0)}_\nu - \frac{1}{2}a^{(2)}_\nu\right) + 2\varepsilon\cos 2\rho a^{(1)}_\nu,$$

$$R^{(\varepsilon)}_{2,\nu} = A^{(2)}_\nu + \sin^2\rho\left(A^{(0)}_\nu - \frac{1}{2}A^{(2)}_\nu\right) - \sin 2\rho A^{(1)}_\nu + 2\varepsilon\sin\rho B^{(1)}_\nu + \varepsilon\cos\rho B^{(2)}_\nu,$$

$$r^{(\varepsilon)}_{2,\nu} = 2\sin\rho b^{(1)}_\nu + \cos\rho b^{(2)}_\nu - \varepsilon a^{(2)}_\nu - \varepsilon\sin^2\rho\left(a^{(0)}_\nu - \tfrac{1}{2}a^{(2)}_\nu\right) + \varepsilon\sin 2\rho a^{(1)}_\nu \; (\varepsilon = \pm 1).$$

As particular case from our inclination functions of corresponding expression of Kinoshita's functions are obtained.

In accordance with generalized Cassini-Colombo laws it inclination is evaluated as ρ= 2'1 on modern data of radiolocation of Mercury. First estimation of this parameter was about 1'6 (Barkin, 1984). Coefficients $A^{(j)}_\nu$, $B^{(j)}_\nu$ and $a^{(j)}_\nu$, $b^{(j)}_\nu$with high accuracy have been presented as quadratic functions of the time which take into account secular planetary perturbations in the Mercury orbital motion (Kudrjavsev, 2009; Barkin, Kudrjavsev, Barkin, 2009): $\mathrm{A}^{(j)}_\nu = \mathrm{A}^{(j)}_{\nu;0} + \mathrm{A}^{(j)}_{\nu;1}\cdot t + \mathrm{A}^{(j)}_{\nu;2}\cdot t^2$, $\mathrm{A} = (A, B, a, b)$, $j = (0, 1, 2)$. These coefficients generalize similar Kinoshita's coefficients (in Earth rotation theory) and represent full and exact developments of following functions of heliocentric spherical coordinates of Mercury (r, φ and λ):

$$\frac{1}{2}\left(\frac{a}{r}\right)^3\left(1 - 3\sin^2\varphi\right) = \sum_\nu A^{(0)}_\nu\cos\Theta_\nu + a^{(0)}_\nu\sin\Theta_\nu,$$

$$\left(\frac{a}{r}\right)^3\cos^2\varphi\cos 2\left(\lambda - h\right) = \sum_\nu A^{(2)}_\nu\cos\Theta_\nu + a^{(2)}_\nu\sin\Theta_\nu,$$

$$\left(\frac{a}{r}\right)^3\cos^2\varphi\sin 2\left(\lambda - h\right) = \sum_\nu B^{(2)}_\nu\sin\Theta_\nu + b^{(2)}_\nu\cos\Theta_\nu,$$

$$\left(\frac{a}{r}\right)^3\sin\varphi\cos\varphi\sin\left(\lambda - h\right) = \sum_\nu A^{(1)}_\nu\cos\Theta_\nu + a^{(1)}_\nu\sin\Theta_\nu,$$

$$\left(\frac{a}{r}\right)^3\sin\varphi\cos\varphi\cos\left(\lambda - h\right) = \sum_\nu B^{(1)}_\nu\sin\Theta_\nu + b^{(1)}_\nu\cos\Theta_\nu.$$

The new expansions are valid over 2000 years, 1000AI 3000AD, have a form similar to that of Kinoshita's series. The latest long-term numerical ephemerides of the Moon and planets DE-406 are used as the source of disturbing bodies coordinates. The mentioned developments have been constructed not only for the problem about Mercury rotation but also for the problems about Earth rotation, Venus rotation and in theory of the Moon rotation (Kudrjavsev, 2009; Barkin, Kudrjavsev, Barkin, 2009). Corresponding developments of Kinoshita in the Earth rotation theory are obtained as particular case from above mentioned formulae by restricting conditions: $r = a = b = 0$.

In the work we analize and evaluate amplitudes, frequencies and phases of solar tidal variations of coefficients of second harmonic of gravitational potential of Mercury. Also tidal perturbations of the Mercury axial rotation caused by variations of polar moment of inertia are determined and analized. The Barkin's work partially was financially accepted by Spanish grants, Japanese-Russian grant N-09-02-92113-JF and by RFBR grant N 08-02-00367.