



DERIVATION OF THE ϵ EQUATION FROM A TWO-POINT CLOSURE

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We present a derivation of the equation for the turbulence dissipation rate ϵ for a shear driven flow. In 1961, Davydov used a one-point closure model to derive the ϵ -equation from first principles but the final result contained undetermined terms and thus lacked predictive power. In 1987, Schiestel and in 2001, Rubinstein and Zhou attempted to derive the ϵ -equation also from first principles using a two-point closure but their method relied on a phenomenological assumption. The standard practice has thus been to employ a heuristic form of the ϵ -equation that contains three empirical ingredients: two constants c_1 , c_2 and a diffusion term D_ϵ . We have employed a two-point closure and obtained the following results: 1) the empirical constants get replaced by new c_1 , c_2 that are now functions of K and ϵ , 2) c_1 and c_2 are not independent since we derive a general relation between the two valid for any K , ϵ , 3) homogenous flows: c_1 , c_2 become constant with values close to the empirical values, 4) inhomogeneous flows: the empirical form of the diffusion term D_ϵ is no longer needed since it gets substituted by the $K - \epsilon$ dependence of c_1 and c_2 which plays the role of the diffusion, together with the diffusion of the turbulent kinetic energy D_K which now enters the new ϵ equation. Thus, the three empirical ingredients c_1 , c_2 and D_ϵ are substituted with a single function $c_1(K, \epsilon)$ or $c_2(K, \epsilon)$, plus a D_K term. We present three tests of the new equation for ϵ , one concerning channel flow and two concerning the shear-driven PBL (planetary boundary layer).