



## Experiments on nonlinear coastal shelf waves in a rotating annulus

Andrew Stewart (1), Paul Dellar (1), and Ted Johnson (2)

(1) OCIAM, Mathematical Institute, University of Oxford, U.K. (stewart@maths.ox.ac.uk, dellar@maths.ox.ac.uk), (2) Department of Mathematics, University College London, U.K. (e.johnson@ucl.ac.uk)

In many coastal regions, the ocean depth increases very rapidly at a “shelf break” running approximately parallel to the coastline. A shelf break marks the edge of the continental shelf, and separates the deep ocean from the relatively shallow near-coastal ocean. Shelf breaks play an important rôle in steering coastal currents, such as the Agulhas current which flows southwest along the eastern coast of Africa at speeds of up to  $1 \text{ ms}^{-1}$ . To investigate the effect of shelf breaks in stabilising coastal currents, we have carried out laboratory experiments to generate nonlinear topographic Rossby waves that propagate along a shelf break in the presence of a mean current.

Our experiments use an annular channel in a rotating cylindrical tank. We model the shelf break with a tank floor that undergoes a sharp drop at a certain radius  $R_h$ . The tank was filled with homogeneous fluid, and set rotating with constant angular velocity until the fluid inside rotated as a solid body. We then induced horizontal perturbations to the fluid, which caused Taylor columns to move inwards and outwards across the shelf. Conservation of potential vorticity forces these columns to acquire relative vorticity as they cross the shelf, which allows waves to propagate around the tank. These waves are known as topographic Rossby shelf waves.

The large-scale flow around shelf breaks has been the subject of a series of theoretical investigations. These commonly approximate the sharp drop in the depth by a discontinuity, on the assumption that the horizontal length scale of the flow is much larger than the width of the shelf break. However, the fluid is still assumed to move in columns, as in shallow water theory, even as it crosses the shelf. Our present work aims to consolidate a theoretical model for nonlinear waves propagating along a depth discontinuity in the context of our laboratory experiments. We assume that rotational effects are dominant, and that fluid velocities are small compared with the surface gravity wave speed. The system may thus be described using the shallow water quasigeostrophic equations with a rigid lid,

$$\frac{Dq}{Dt} = 0, \quad q = \omega + \frac{fh}{H}, \quad \omega = \nabla^2 \psi.$$

Here  $q$  is the potential vorticity,  $\omega$  is the two-dimensional relative vorticity,  $\psi$  is the streamfunction,  $H$  is the maximum height of the fluid,  $f$  is the Coriolis parameter,  $h$  is the spatially-varying height of the bottom topography, and  $D/Dt$  is the advective derivative. These equations have previously been applied to study topographic Rossby shelf waves in a straight channel with a discontinuity in the depth part-way across. By looking for solutions that vary slowly along the channel, a nonlinear wave equation has been derived to describe the evolution of a potential vorticity front that lies initially along the discontinuity in depth.

To make predictions about the waves generated in our experiments, we have reformulated this existing theory for straight channels to describe long nonlinear topographic Rossby shelf waves in an annular channel. However, we find that the resulting theoretical predictions are not verified by the experiments, in which short-wave disturbances grow rapidly and dominate the large-scale flow. Being based on finding solutions that vary slowly around the channel, the long-wave theory cannot capture the behaviour seen in the experiments.

Motivated by this discrepancy, we have conducted a numerical study of the shallow water quasigeostrophic equations given above, without making any further assumptions about the wavelengths of the disturbances. These calculations suggest that short-wavelength phenomena with large amplitudes will always appear, even in an initial flow configuration of very long wavelength, as long as the amplitude of the initial disturbance is large enough for the waves to be nonlinear. However, the earlier long-wave theory still gives a qualitatively accurate description of the evolution of very long waves.