Calculating the external magnetic force on the Earth’s dipole

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The force exerted by the external perturbation magnetic field on the magnetic moment of a planet is related to various aspects of stress balance in the magnetosphere and its interaction with the solar wind. The total force applied by the solar wind must ultimately be exerted on the planet itself, which contains essentially all the mass of the entire system (the mass fraction in the Earth’s magnetosphere is less than $\sim 10^{-20}$ of the total). The force is transmitted through the magnetosphere primarily (and near the planet almost exclusively) by the magnetic field. In the simplest approximation (Siscoe, 1966), the force is applied directly as the gradient of the external perturbation field at the dipole. More recently (Siscoe and Siebert, 2006; Vasyliunas, 2007), it has been recognized that coupling by Birkeland currents between the ionosphere and the magnetosphere allows the external force to be transmitted also as a $J \times B/c$ force in the ionosphere; further transmission to the planet itself then has to proceed as a mechanical stress. Because of the converging dipole field, the force in the ionosphere is greatly amplified and much stronger than the initially imposed force from the magnetosphere or solar wind, an effect sometimes described as the mechanical advantage of the magnetosphere (Vasyliunas, 2007). Empirical estimates of the force thus provide a sensitive (albeit indirect and imprecise) indicator of stresses in the outer magnetosphere, as well as a direct measure of the global input of linear momentum into the atmosphere. The total magnetic force on the planet can be calculated from measurements of magnetic perturbations by integrating the Maxwell stress tensor over the surface. I derive the formula for the three vector components of the force in terms of the conventional geomagnetic quantities, integrated over latitude and longitude with appropriate weighting factors (which are not always intuitively obvious and in some cases reverse sign between low and high latitudes).