



Pullback attractors in nonautonomous dynamical systems with delay: Applications to an ENSO model with seasonal forcing

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Recent work by two of the co-authors (MDC and MG) on random dynamical systems and their attractors has motivated us to consider also a closely related problem, namely the pullback attractors (PBAs) of deterministic dynamical systems driven by time-dependent forcing. Two obvious examples of such forcing in climate dynamics are: (i) periodic forcing by the seasonal cycle; and (ii) slow forcing by an interdecadal warming trend.

We illustrate these two types of forcing in a highly idealized model for El Nino/Southern Oscillation (ENSO) variability, previously studied by two of the co-authors (MG and IZ). The model is governed by a delay differential equation (DDE) for sea surface temperature T in the Tropical Pacific, and it combines two key mechanisms that participate in ENSO dynamics: delayed negative feedback and seasonal forcing. We perform a theoretical and numerical study of the model in the three-dimensional space of its physically relevant parameters: strength of seasonal forcing, atmosphere-ocean coupling, and propagation period of oceanic waves across the Tropical Pacific. We report several findings that are consistent with the observed dynamics of ENSO, as well as with more detailed and realistic models: quasi-periodic behavior with the correct “period” values, phase locking, and sensitive dependence on model parameters. We demonstrate, furthermore, that the transition from smooth period-1 behavior to more realistic — quasi-periodic or chaotic — behavior occurs in a sequence of bifurcations similar to those observed in classical chaotic systems.

We study next a multi-dimensional manifold of solutions, given by constant or piecewise constant initial histories, e.g. a constant, year-long warm (El Nino) or cold (La Nina) state. To do so, we apply the PBA concept in order to study the model dynamics in the model-parameter region where sensitivity to parameter values is high, as well as in the transition region from smooth to sensitive behavior. Computation of the model’s PBAs clearly demonstrates that its dynamics — whether periodic (smooth) or quasi-periodic (sensitive) — occurs on a two-dimensional torus. This behavior reflects the competition between two oscillatory mechanisms: an external one due to the seasonal forcing and an internal one due to the delayed feedbacks. Such an interpretation is much harder to obtain from the complex, parameter-sensitive dynamics of the model using more traditional approaches for autonomous systems.

Finally, we study the model dynamics subject to long-term global warming scenarios and interpret the results in the context of the ongoing research on long-term climate projections.