



## The use of spatio-temporal correlation to forecast critical transitions

Derek Karssenberg and Marc F. P. Bierkens

Department of Physical Geography, Faculty of Geosciences, Utrecht University, Utrecht, the Netherlands  
(d.karssenberg@geo.uu.nl)

Complex dynamical systems may have critical thresholds at which the system shifts abruptly from one state to another. Such critical transitions have been observed in systems ranging from the human body system to financial markets and the Earth system. Forecasting the timing of critical transitions before they are reached is of paramount importance because critical transitions are associated with a large shift in dynamical regime of the system under consideration. However, it is hard to forecast critical transitions, because the state of the system shows relatively little change before the threshold is reached. Recently, it was shown that increased spatio-temporal autocorrelation and variance can serve as alternative early warning signal for critical transitions. However, thus far these second order statistics have not been used for forecasting in a data assimilation framework.

Here we show that the use of spatio-temporal autocorrelation and variance in the state of the system reduces the uncertainty in the predicted timing of critical transitions compared to classical approaches that use the value of the system state only. This is shown by assimilating observed spatio-temporal autocorrelation and variance into a dynamical system model using a Particle Filter.

We adapt a well-studied distributed model of a logistically growing resource with a fixed grazing rate. The model describes the transition from an underexploited system with high resource biomass to overexploitation as grazing pressure crosses the critical threshold, which is a fold bifurcation. To represent limited prior information, we use a large variance in the prior probability distributions of model parameters and the system driver (grazing rate).

First, we show that the rate of increase in spatio-temporal autocorrelation and variance prior to reaching the critical threshold is relatively consistent across the uncertainty range of the driver and parameter values used. This indicates that an increase in spatio-temporal autocorrelation and variance are consistent predictors of a critical transition, even under the condition of a poorly defined system.

Second, we perform data assimilation experiments using an artificial exhaustive data set generated by one realization of the model. To mimic real-world sampling, an observational data set is created from this exhaustive data set. This is done by sampling on a regular spatio-temporal grid, supplemented by sampling locations at a short distance. Spatial and temporal autocorrelation in this observational data set is calculated for different spatial and temporal separation (lag) distances. To assign appropriate weights to observations (here, autocorrelation values and variance) in the Particle Filter, the covariance matrix of the error in these observations is required. This covariance matrix is estimated using Monte Carlo sampling, selecting a different random position of the sampling network relative to the exhaustive data set for each realization. At each update moment in the Particle Filter, observed autocorrelation values are assimilated into the model and the state of the model is updated.

Using this approach, it is shown that the use of autocorrelation reduces the uncertainty in the forecasted timing of a critical transition compared to runs without data assimilation. The performance of the use of spatial autocorrelation versus temporal autocorrelation depends on the timing and number of observational data.

This study is restricted to a single model only. However, it is becoming increasingly clear that spatio-temporal autocorrelation and variance can be used as early warning signals for a large number of systems. Thus, it is

expected that spatio-temporal autocorrelation and variance are valuable in data assimilation frameworks in a large number of dynamical systems.