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Tokunaga self-similar trees: A characteristic property of aggregation processes

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Hierarchical branching organization is ubiquitous in nature. It is readily seen in river basins, drainage networks, bronchial passages, botanical trees, and snowflakes, to mention but a few. Empirical evidence suggests that one can describe many natural hierarchies by so-called Tokunaga self-similar trees (SSTs); Tokunaga SSTs form a special class of SSTs that preserves its statistical properties under the operation of pruning, i.e., cutting the leaves.

Why do Tokunaga trees emerge so often? We conjecture that Tokunaga self-similarity is a characteristic property of the inverse aggregation (coagulation) process. To support this claim, we develop a statistical test for Tokunaga self-similarity and analyze numerically three generic aggregation phenomena: (1) nearest-neighbor clustering in n-dimensional Euclidean space with n = 1,...,100; (2) topological structure of the level sets of a fractional Brownian motion whose Hurst index H satisfies $0 \leq H \leq 1$; and (3) N-point clustering with preferential attachment. It is shown that (i) all three phenomena are closely approximated by Tokunaga SSTs; and (ii) they reproduce a broad range of Tokunaga self-similarity parameters. Our numerical results are in agreement with existing theoretical results, which are only available for regular — i.e., not fractional — Brownian motion.

We furthermore proceed with an analysis of self-similar properties of so-called dynamic trees recently introduced by the authors in the study of river networks. A dynamic tree describes transport of fluxes from the leaves to the root of a static tree. It is thus very useful for investigating environmental fluxes of water, sediment or pollutants. We show numerically that a dynamic tree constructed on a static Tokunaga SST is also Tokunaga SST, albeit with different values of its self-similarity parameters.