



## Waveform inversion in the Laplace and Laplace-Fourier domains

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We present a review of full waveform inversion in the Laplace and Laplace-Fourier domains.

Since Tarantola's pioneering work on waveform inversion, the practical application of full waveform inversions to real field data in the time or frequency domain has been nominal. The main hindrances in recovering a long-wavelength velocity model through full waveform inversion originate from a lack of low-frequency information in the data and the high non-linearity of the objective functions.

The Laplace-domain wave equation can be obtained simply by changing the real frequencies from the Fourier transform into imaginary numbers. In terms of numerical integration, the Laplace-transformed wavefield is the sum of the damped wavefield from zero to infinity. Hence, Laplace-domain wavefields for each Laplace frequency can be treated as zero-frequency components of damped wavefields. The Laplace-transformed wavefield resembles a direct current field where only the source is used for Poisson's equation.

The logarithmic objective function, combined with the Laplace transformed wavefield, has no local minima and a smoother convex form than conventional objective functions in the time or frequency domain. These characteristics of the Laplace domain wavefields and the objective function make Laplace domain inversions robust for real data, allowing us to start from scratch and recover smooth velocity models. Moreover, coarse grids, compared with time or frequency domain numerical modeling, can be used in the Laplace-domain inversion without sacrificing accuracy. The choice of Laplace frequencies to be used for inversion is roughly determined by inspecting a kernel of the Green's function for a horizontally two-layered media.

Just as the zero-frequency component is exploited in a Laplace-domain inversion, a Laplace-Fourier domain inversion utilizes low, medium and high frequency information of damped wavefields. By introducing complex-valued frequencies in the Fourier transform and adjusting the real and imaginary part of these complex frequencies, we can numerically calculate Laplace-Fourier transformed wavefields for several damped wavefields in the time domain. The complex-valued wavefield allows us to use both amplitude and phase information, while in the Laplace domain, only the real-valued wavefield is used for the inversion. Through simultaneous inversion of the amplitude and phase of complex-valued logarithmic wavefields, medium- and short-wavelength velocity structures can be recovered in addition to long-wavelength structures, and the penetration depth of the inversion can be enhanced.

Since the Laplace-domain techniques are based on the damped wavefields, Laplace-domain inversion is very sensitive to noise appearing before the first arrival. Therefore, careful muting of this noise is needed for successful inversion to real data. To make the Laplace transform stable, the maximum recording time should be long enough so that the amplitude of damped signals after the maximum recording time can be ignored within the noise level. In particular, Laplace domain inversion also requires larger offset data than the frequency-domain inversion because the logarithmic residual tends to form a long-wavelength shape.

Inversion results of Laplace and Laplace-Fourier domain inversion can be used as a starting velocity model for subsequent frequency-domain full waveform inversion. This two-step strategy may produce correct high-resolution velocity models from several sources of synthetic and field data. Laplace-domain inversions with a coarse grid make 3-D seismic inversion feasible. Elastic and acoustic-elastic coupled inversions in the Laplace and

Laplace-Fourier domain have the potential to delineate elastic parameters from land and marine data. Data with a large offset and long recording time can produce good inversion results in either the Laplace or Laplace-Fourier domain. The selection of the optimum combination of Laplace-Fourier frequencies, as well as the choice of damping constants for Laplace domain inversions, are not firmly established and need extensive study in the future.