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On the roto-translatory internal motions of a three layer non-isobarycentric Earth model: a Lagrangian system approach

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The internal structure of numerous celestial bodies are well approximated by means of a three layer model composed of a solid external layer, which encloses a fluid layer containing a solid body. An analysis of the inner dynamics of this model can provide some constrains on its rheological characteristics; an information that in many situations is only accessible through this indirect way. In addition, the understanding of this kind of motions, especially of those associated with a rigid displacement (a rotation or a relative translation) of the solid layers, is of primary importance to establish with enough accuracy the definition of the terrestrial reference frames.

In the Earth case, most approaches to this formidable problem rely on the numerical solution of the respective elastic field equations, once they have been projected on a set of spherical harmonics functions of a given degree. Due to its intrinsic nature these numerical methods do not provide by themselves much insight into the internal dynamics, hence the interest to develop simpler dynamical models that reproduces the main characteristics of the motion and allows obtaining analytical approximate solutions of the problem.

To this aim, and as a first stage, we have considered the internal dynamics of a simple Earth model made up of a spherical rigid mantle, an inviscid, homogeneous fluid outer core and a spherical rigid inner core. Initially the barycenters of all the constituents are located at the same point (isobarycentric model) and the whole system rotates with constant angular velocity around the figure axis. When this situation is perturbed both the motions of the fluid and of the solid layers depart from the reference uniform rotation. However, following Busse (1974) we have assumed that the motion of the mantle is the same as in the unperturbed state, and that the inner core dynamics only suffers a variation of oscillatory nature in the translational motion of its barycenter. As a consequence of this relative motion, three characteristic proper modes appear: one in the direction of the figure axis (polar mode) and two orthogonal to it (equatorial modes). These modes are usually referred as Slichter triplet. In the case of the polar mode, Busse (1974) determined analytically its expression in an implicit way; later other authors have obtained by numerical methods the values of all the modes (e.g Rieutord 2002). These expressions differ substantially from the single degenerate mode existing for a non-rotating model, the differences arising from the roto-traslatory coupling of the system.

To construct an analytical description of the motion of this non-isobarycentric Earth model we have approximated it by a Lagrangian system, inspired in the successful of this variational approach to tackle the rotational dynamics of isobarycentric Earth models (e.g. Moritz 1982, Getino and Ferrándiz 2001). In this way, the fluid flow is represented as the sum of a rigid motion part plus a potential motion part. In this way, the resulting dynamical system is described by means of nine generalized co-ordinates. Once constructed the kinetic energy of each layer of the Earth model and the potential energy due to the gravitational interaction of the spherical rigid inner core with the fluid, we form the Euler-Lagrange equations of the system which turn out to be non-linear. By assuming a small departure with respect to the steady rotation configuration we linearice the differential equations of the motion, deriving from them the analytical expressions of the Slichter triplet. These expressions are compared with the existing numerical ones, appearing some discrepancies between both approaches. They might be caused by neglecting the non-linear terms in the resolution of the equations or by an incomplete description of the fluid flow. However, the numerical values of the modes derived with this treatment show a great improvement with respect to the values obtained from performing

simply a kinematical correction on the degenerate mode of a non-rotating model. Namely, the relative errors in the periods of the equatorial modes are reduced from 11.69% to 3.79% and from -6.04% to -1.91%, respectively.

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