



## **Handling nonlinearity in two nonlinear Kalman filters with symmetric analysis ensembles**

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The Kalman filter (KF) is a sequential data assimilation scheme that achieves the optimal performance in linear Gaussian systems. In the presence of nonlinearity, several methods have been developed to allow the implementation of the KF. We refer to the resulting filters as nonlinear Kalman filters, which include, for example, the extended Kalman filter (EKF), the ensemble Kalman filter (EnKF), and the filters discussed below. Roughly speaking, it is the ways in handling nonlinearity that make the nonlinear KFs distinct from each other.

Here we focus on two nonlinear KFs, called the unscented Kalman filter (UKF) and the divided difference filter (DDF), respectively. Although these two filters are developed based on different philosophies, they share the same analysis scheme. More concretely, each filter generates as the analysis ensemble some special system states (called sigma points), which are symmetric about the analysis mean and preserve the analysis covariance.

The UKF is an EnKF-type filter, which adopts Monte Carlo approximations to estimate the mean and covariance of the system states. The main feature of the UKF is that its analysis ensemble is symmetric about the analysis mean. It can be shown that, with the symmetry, the UKF can avoid some of the sampling errors and biases in the EnKF.

The idea of the DDF is similar to that of the EKF. Like Taylor series expansion used in the EKF, Stirling's interpolation formula is adopted in the DDF to expand a nonlinear function locally. Through the formula, there is no need for the DDF to evaluate the derivative(s) of a nonlinear function. Here the analysis ensemble serves as interpolation points, which are required to be symmetric about the local point (e.g., the analysis mean) where the nonlinear function is expanded.

In this contribution we discuss the benefits of symmetric sampling and compare the theoretical and practical aspects of the UKF and DDF. We suggest strategies to reduce the computational costs of both filters in applications. Numerical results will be presented and discussed.