



On estimate of the regular region boundaries in a singular vortical model under finite nonstationary perturbation

Evgeny Ryzhov and Konstantin Koshel

V.I. Il'ichev Pacific Oceanological Institute of FEB RAS, Vladivostok, Russia (ryzhovea@poi.dvo.ru)

The chaotic advection of passive particles in a vortex, generating by a topographic background current, is analyzed [1-2]. The concept of the background currents [1-3] represents a wide class of vortical geophysical flow models admitting chaotic solutions. We consider such two-layer model, giving an opportunity to take into consideration the simplest example of stratification of fluid [3].

It is known, that the regular advection region always exists in vicinity of a singular elliptic point for any perturbations, including finite ones. The question why trajectories are regular in the vicinity of the singular point, though they all are unstable from the linear approach is also opened. On the one hand, knowing of the regular advection boundaries is useful for understanding of a passive particle transport in the model [3]. On the other hand, this estimate is usable to the oceanology in order to interpret point vortices as finite-core ones [1].

A positive Lyapunov exponent is known, results in the manifestation of chaotic properties of trajectories [3]. It is suggested to estimate the boundaries of regular behavior of trajectories by using the Lyapunov vector projection to the normal of a trajectory.

The trajectory divergence, be calculated as the sum of the absolute value of the Lyapunov vector projections with respect to its sign toward the normal, is supposed to give the boundary of the regular motion.

Inside the regular advection region this divergence tends to zero, caused by changing the projection sign. Whereas, inside the chaotic region the projection sign changes non-uniformly, resulting in distinct growth of the divergence. Thus, this growth shows the chaotic dynamics region.

As the criterion to determine the maximum boundary of the regular region in vicinity of a singular vortex, we suppose to use the Chirikov criterion, known also as the criterion of nonlinear resonances overlapping [3].

We show that in dynamics systems with a singular stagnation point, the Chirikov criterion of resonances overlapping tends to zero in vicinity of this point [3]. Thus, the criterion answers the question about existence of the regular region in vicinity of the singular point when the perturbation is not only small at all, but also is finite. It means that the widths of the nonlinear resonances are decreasing more rapidly than the distances between the closest ones. Hence, the overlapping between these closest resonances does not exist, and all trajectories remain nonchaotic in vicinity of the singular point of the vortical movement.

In order to improve the precision of the determination of the regular region, we use more precision criterion for nonlinear resonances of higher multiplicativity, which yields a plausible estimate of the regular region border [3]. Also this more precision criterion gives an opportunity to determine the optimal frequencies interval. The comparison between the optimal frequencies interval, obtained by the calculation of vanished particles from the vortical region, and the optimal frequencies interval, obtained by the help of the more precision criterion, shows appropriate correspondence.

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