



Scattering of Internal Waves with Frequency Change over Rough Topography

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The frequency change in internal gravity waves upon scattering from a rough topography is investigated analytically. For this, sets of appropriate and tractable governing equations for various parameter regimes are derived using the method of multiple scales under the assumption that the amplitude of the bottom topography is small. A solution is shown for a simple case in which an incident internal wave is approximately linear and monochromatic. The solution has the following features. The intrinsic frequencies of the scattered waves are given as the sum and difference of the incident-wave frequency and the Doppler shift (or lee-wave frequency). This Doppler shift causes the change in the frequency. Hence, the assumption of frequency conservation is not valid if the Doppler shift is significant, i.e., when the horizontal scale of the bottom roughness (or the length scale in the plane of the slope) is of the order of or much less than that of the incident-wave flow excursion. This condition can be satisfied in a realistic parameter range.

The occurrence of such a frequency change has the following implications. Firstly, it affects the estimate of the boundary mixing induced by the scattering since the energy redistribution in the vertical wavenumber space on scattering differs from that estimated using the assumption of frequency conservation. This effect is caused because for a given horizontal wavenumber, the change in the frequency alters the vertical wavenumber of the scattered waves through the dispersion relation. Furthermore, if the incident waves are not monochromatic, even the leading-order scattered waves cannot be obtained by the superposition of the solutions for all the Fourier components of the incident waves due to the difference in the Doppler shift. Secondly, the effects of the background flow associated with the incident and primary reflected waves are significant when the frequency change occurs such that the background flow can create a critical level and/or advect scattered waves. The former causes mixing and background-flow acceleration and the latter is favorable for the amplification of the scattered waves through superposition. Thirdly, the resulting energy redistribution in frequency space could modify the spectrum shape of the oceanic internal waves, which is considered to affect both interior and boundary mixing.