



## **Nonlinear development of unstable wave-modes coupled in the critical layer of a weakly supercritical zonal flow**

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This study explores nonlinear development of barotropic instability in a horizontally sheared quasigeostrophic zonal flow on the beta-plane in the presence of the stable atmospheric density stratification. The background zonal flow is specified by a free shear layer velocity profile most typical of the Antarctic polar vortex periphery. According to the linear inviscid theory based on the Rayleigh-Kuo's theorem free shear layer flows are capable of supporting both baroclinic and barotropic unstable modes whenever the gradient of the Coriolis parameter  $\beta$  is smaller than a marginal value  $\beta_m$ . Supercriticality of the inviscid flow defined as  $\delta\beta = \beta_m - \beta$  is set to be of the order of a small parameter  $\varepsilon$ . To capture effects arising from small dissipation an inverse Reynolds number and an Ekman dissipation parameter are also scaled in terms of  $\varepsilon$ . Extracting the kinetic energy of the basic flow by simultaneously growing weakly unstable Rossby wave-modes in a weakly dissipative zonal flow near the onset of instability is confined to their common nonlinear critical layer (CL) (relatively thin resonant region surrounding the level where the wave speed of marginal modes matches the mean flow). Only two unstable modes coupled in the CL (the barotropic mode and the main baroclinic one) are shown to develop in the flow provided that some restriction is imposed on the internal deformation radius value.

For this case an asymptotic approach based on the method of the matched asymptotic expansions in  $\varepsilon$  is employed to derive a closed system of equations governing the dynamics of the wave-modes amplitudes and evolution of the potential vorticity in the nonlinear CL. If the supercriticality is small enough for dissipation to dominate inside the CL, the CL-flow evolves in a quasi-steady weakly nonlinear regime and an approximate solution for the potential vorticity in the form of an expansion in the small wave amplitudes can be obtained. In this case simultaneous development of the wave-modes is governed by a reduced system of two coupled Landau-Stuart equations. In the absence of resonant modes coupling through the condition of the second harmonic resonance the primary effect of their nonlinear interaction inside the CL is the suppression of the baroclinic mode. The physical mechanism of this effect is based on the energy transfer from the baroclinic mode to the barotropic one caused by the interaction of the wave-modes with the second-order beat-wave vorticity perturbations in the CL. At the higher levels of the supercriticality nonlinearity and time dependence play a significant role in the CL and development of the instability can no longer be described by the weakly nonlinear equations. For this reason the flow regimes and instability development scenarios arising successively as the supercriticality is increased are studied with the aid of a numerical scheme including the Fourier series expansion in zonal direction and the finite-difference approximation across the CL for the periodic boundary conditions corresponding to the annular zonal flow. In particular for the moderate levels of the supercriticality mutual suppression of the barotropic and the baroclinic modes (modes competition) leading to the modes selection is revealed.