



## Application of 3-dimensional copula in the estimation of low flow extremes

Wojciech Jakubowski

Department of Mathematics, WROCLAW UNIVERSITY OF ENVIRONMENTAL AND LIFE SCIENCES, Poland  
 (Wojciech.Jakubowski@up.wroc.pl)

Consider the low flows obtained by POT method. Each received low flow can be described by three indices: deficit -  $D$ , duration -  $T$  and lowest outflow -  $M$ . For estimating the distribution of triple  $(D, T, -M)$  the three-dimensional Gumbel - Hougaard coupla

$$C(u_1, u_2, u_3) = \exp \left\{ - \left[ (-\ln u_1)^\phi + (-\ln u_2)^\phi + (-\ln u_3)^\phi \right]^{\frac{1}{\phi}} \right\} \quad (1)$$

is applied. In (1):

- parameter  $\phi$  defines the relationship between investigated indices;
- $\varphi(u) = (-\ln u)^\phi$  is a generate function.
- $u_i, i = 1, 2, 3$  are uniformly distributed random variable varying from 0 to 1;
- $u_i = F_i, i = 1, 2, 3$  are marginals of trivariate random variables.

It is assumed that the marginal distributions are one dimensional extreme values distributions GEV:

$$F_i(x) = \exp \left\{ - \left[ 1 + \xi_i \left( \frac{x - \mu_i}{\sigma_i} \right) \right]^{-\frac{1}{\xi_i}} \right\}, \quad x > \mu_i - \frac{\sigma_i}{\xi_i}, \xi_i > 0 \quad (2)$$

with unknown parameters  $\mu_i, \sigma_i$  and  $\xi_i$ .

Estimation of copula distribution (1) is carried out in two steps:

1. Unknown parameters of each of the distributions (2) are estimated by maximum likelihood method;
2. For the first step received distributions  $F_i$  parameter  $\phi$  is also estimated by maximum likelihood method.

To examine the goodness of fit of the estimated GEV distributions (2) as well as copula (1) with the observed indices the test  $\chi^2$  is applied.

The result shows the one-and two-dimensional distributions of low flow deficit and the duration, depending on minimal outflow. This means that the following probabilities are determined:

$$\Pr(D \leq x, T \leq x | M > q) \quad (3)$$

where  $q$  is chosen minimal outflow.