



The Perturbation Analysis of Nonlinear Ill-conditioned Solution

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Generally, the ill-conditioned problem not only exists in geoscience, such as global positioning system(GPS) data processing, geophysical inversion, transformations for modern terrestrial reference frame, et al., but also its effect are very serious. When ill-conditioned problems are encountered, it is of great significance to analyze the deep-seated causes for weakening the ill-conditioned.

To get a solution of practical problems, we need models construction, observation, the selection of the minimization criteria and solution. Also, the study of ill-conditioned problems is affected in the four aspects. Usually, we deal with linear models or an approximate linear model through the Taylor expansion, and take Least Square(LS) criteria. In this particular case, the ill-conditioned nature of the data, the parameter estimate and the minimization criterion are the same, ultimately reflecting in the multicollinearity of the matrix.

However, if the model is nonlinear and the nonlinearity is high, it is very sensitive to the choice of approximation value. In addition, with development of the observation techniques, the observation accuracy has been greatly improved, and then the error generated by the linear approximation of the nonlinear model is considerable to the observation error, some even more than the observation error.

In addition to the ill-conditioned analysis of the nonlinear model itself, the perturbation stemmed from truncation and orthogonalization process needs to be considered. In this study, the perturbative estimation inequality of nonlinear ill-conditioned problem is derived from the definition of the generalized condition number. And based on the perturbative estimation inequality, the impacts of judgement and analysis are studied on nonlinear ill-conditioned problem. The impacts stem primarily from the disturbance of the coefficient matrix of linear approximation and additional truncation error, also including orthogonal approximation. Therefore, nonlinear ill-conditioned problem is verified and analyzed through two examples. The results of the research show that: when the model nonlinearity is strong, the disturbance of the coefficient matrix and additional truncation error are very significant due to the selection of different approximation values and orthogonal approximation. The large disturbance and truncation error bring results variability of the ill-conditioned problems. In this case, iterative regulation method can be used to solve nonlinear ill-conditioned problems.

Key words: nonlinear ill-conditioned problems; the generalized condition number; perturbation analysis; linearization; orthogonalization