



About the angles of inclination of the rotational axis and the angular momentum of Mercury

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Abstract. The paper shows the ascending node of equator of Mercury (and the intermediate plane orthogonal to the angular momentum) of epoch 2000.0 on the ecliptic does not coincide with the ascending node of orbital plane of Mercury on the same plane, and ahead it at an angle $23^{\circ}4$. Angular momentum vector of the rotational motion of Mercury form a constant angle $\rho_G = 4'1 \pm 1'1$ with normal to the moving plane of the orbit. The observed inclination of the angular velocity $\rho_\omega = 2'1 \pm 0'1$, possible evidence of a significant amplitude free movement of the poles of the rotation axis of Mercury (c amplitude of about $2' - 3'$), as predicted in [1].

Model. Resonant motion of Mercury on Cassini have been studied well-known authors: Colombo, Beletskii, Peale, etc. Moreover, as a rule for the perturbed orbital motion was taken on the motion of a uniformly precessing orbit (with a constant angular velocity $n_\Omega < 0$) with constant angle of inclination of orbit plane i relatively the base plane (ecliptic plane or Laplace plane). The orbit is elliptical and is characterized by constant eccentricity e . In this study, the rotational motion of Mercury (as a celestial body with nonspherical solid mantle and liquid core) on an evolving orbit, referred not to the Laplace plane and the ecliptic of the given epoch. We take into account not only the uniform precession of the orbit plane (the secular change in longitude of the ascending node of the orbit Ω), but the slow change in orbital inclination (i) with small angular velocity n_i .

The base model of Mercury's orbit in the study of its rotational motion in this paper take the mean orbit of this planet, whose parameters are given in the famous website <http://ssd.jpl.nasa.gov/?planets#elem>: Planetary Mean Orbits (J2000) (epoch = J2000 = 2000 January 1.5):

$$i = 7^{\circ}0028806, \quad n_\Omega = \frac{d\Omega}{dt} = -446''30 \text{ (1/cy)}, \quad n_i = \frac{di}{dt} = -23''57 \text{ (1/cy)}.$$

Period of orbital motion of Mercury is $T_n = 2\pi/n = 87.969$ days (n in the mean orbital motion) and period of progressive precession of the line of node of the orbit plane on the Laplacian plane is $T_\Omega = 2\pi/|n_\Omega| = 278898$ years.

An estimation of the value of the angle of inclination of the angular momentum relative to the normal to the mean orbital plane ρ_G , is made on the basis of non-normalized values of the coefficients of the second harmonic of the gravitational field of Mercury J_2 and C_{22} (gravity model HgM001 MESSENGER) [2]: $J_2 = (1.92 \pm 0.65) \cdot 10^{-5}$, $C_{22} = (0.81 \pm 0.08) \cdot 10^{-5}$, on parameters of physical librations and internal structure of this planet, derived from satellite observations by apparatus Messenger, ground-based radar observations [3]: $(B-A)/C_m = (2.03 \pm 0.12) \cdot 10^{-4}$, $\rho_\omega = 2'1 \pm 0'1$ and based on theoretical estimates [4]: $C/(mR^2) = 0.35$, $C_m/C = 0.5 \pm 0.07$. Here C and C_m are polar moments of inertia of Mercury and its mantle. m and r are mass and mean radius of Mercury.

Cassini's motion of Mercury. Based on the method of investigation of resonant rotational motion of Mercury, developed in [5], [1], for considered here the model of an evolving orbit, we obtain the following analytical expressions and numerical values of the generalized Cassini's motion of Mercury (for the unperturbed values of the ascending node h_0 and inclination angle ρ_G of the vector angular momentum of the planet relative to the ecliptic of 2000.0):

$$h_0 = \arctan(-n_i/n_\Omega \sin i) = 23^{\circ}3677,$$

$$\rho_G = -\frac{n_\Omega \sin i}{n_0 \cosh_0} \left[\frac{n_\Omega \cos i}{n_0} + \frac{1}{I} \left(J_2 C_0^{(-3.0)} + 2C_{22} X_N^{(-3.2)} \right) \right]^{-1} = 4'2 \pm 1'4.$$

From these formulas it follows that due to influence of the angle h_0 the value of Cassini's angle ρ_G increases at 8.94%, compared with an earlier value for $h_0 = 0$ [1], [5].

Conclusion. If the secular change in inclination of the orbit of Mercury to exclude from consideration, then these equations can be simplified and its solution will be $h_0 = 0$ and $\rho_G = 3'9 \pm 1'3$. That solution with $h_0 =$

0 (for others values of Mercury parameters) before, starting with the pioneering works of Colombo, Beletskii, Peale et al., have been studied actively. We emphasize here that the rotational motion is not attributable to the Laplace plane for the Mercury, and with respect to the mean ecliptic and equinox of epoch 2000.0. In [1] we have suggested the existence of large amplitude free librations of Mercury in longitude with a period of 12 years. As the main mechanism of excitation of free oscillations was proposed mechanism of forced relative oscillations of the core and mantle of the planet (which are non-spherical bodies and occupy the eccentric positions relative to each other) under the gravitational attraction of the Sun and the planets and due to perturbations in orbital motion. Naturally, this same mechanism is responsible for the excitation of free motion of the pole and the free oscillations of angular momentum vector in space. The results of this study can be regarded as a confirmation of the existence of long-period (not Euler) oscillations of the rotation axis of Mercury with an amplitude of about $2'' - 3''$. And if the prediction of the free librations in longitude [1] has already received confirmation [3], to confirm the free oscillations of the pole axis of rotation of Mercury in the body and the free oscillations of angular momentum vector in space should be a new and more accurate data on the gravitational field of Mercury and its librations. This issue should contribute to research on the MESSENGER spacecraft on mercurial orbit in 2011. The action of the mechanism of forced swing and wobble of the core and mantle of Mercury (and of others solar system bodies) should lead to the formation of active geological structures with an asymmetric distribution with respect to the northern and southern hemisphere and especially pronounced in polar regions. The first position was clear evidence in the asymmetric distribution of scarps and ridges of Mercury. We expect that the second assumption will be confirmed in studies of the polar regions of the MESSENGER spacecraft in March-April 2011. In conclusion the remark that the phenomenon is revealed shift of the lines of nodes $h_0 = 23^{\circ}4'$ can make some adjustments in the calculation according to radar observations of Mercury's rotation carried out in [3].

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