



Unstructured Grid Discontinuous Galerkin Methods for Glacial Modeling at Large Space and Time Scales

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We will discuss a new numerical strategy that we have applied to the simulation of glacial evolution over large space- and time-scales. The initial application derives from a thermomechanical shallow-ice based approximation that previously appeared in the University of Toronto Glacial Systems Model (GSM). GSM physics are invoked in the context of a new code-base that we developed from scratch in order to introduce state-of-the-art unstructured grid (UG) finite element (FE) numerical techniques. The rationale for such work is in the need for better mathematical and computational methods for representing the complex approximations arising in ice-sheet modeling. The new GSM is intended to serve as a versatile framework and test-bed for comparing the implications of differing physical assumptions. Previously published results from the original GSM focus on glaciation in paleoclimate scenarios, and the new work continues to address this important topic. For ease of comparison with other techniques, paleoclimate test cases are derived from the simple EISMINT ansatz for the simulation of the glacial history of Greenland.

The rationale for UG and FE methods arises because the problem of land-ice simulation is complicated by well-known multi-scale characteristics that arise in a variety of ways. For instance, the bulk dynamics of glaciers at large scales are disproportionately influenced by their interactions with orography and/or seawater at narrow, geometrically complex ice margins. For less obvious reasons, interior mass transport is also strongly affected by narrow ice-streams that move much more quickly than surrounding ice. Such factors, along with the practical localization of available observational data, motivate the use of UG-FE grids in which resolution can be arbitrarily localized. Our new GSM framework is distinguished by the fact that it goes beyond the class of FE methods that have hitherto been considered for land-ice-modeling.

In experimenting with numerical methods, we aim to deduce the representation that is most naturally suited to land-ice dynamics. On purely logical grounds, the physics must under any circumstances fit a conservation problem in which the transport (flux) derives from the properties of the ice. The “standard” continuous Galerkin, or CG, methods that are featured in introductory FE textbooks are well-suited to the implicit solution of diffusion-dominated (parabolic) flux-conservation problems. Since the isothermal limit of the shallow-ice equations yields a pure nonlinear diffusion equation, it might be presumed that there is an open-and-shut case for the use of CG methods. The basic equations can, however, be equivalently analyzed in terms of advective transport, and higher-order ice dynamics must almost certainly exhibit important advection-dominated (hyperbolic) components. Hyperbolic dynamics are best represented by relatively newer and less well-known discontinuous Galerkin (DG) methods, and we have adopted these as the basis for the new framework.

Although both CG and DG methods are adequate in principle for land-ice modeling, we believe that the latter is more natural, especially as physical approximations become more complex. Inter-comparisons of methods for various test-cases will in any case yield insights into the complexities of land-ice dynamics. The theory and results presented will address these issues.