



## A Statistical Mechanical Approach for the Computation of the Climatic Response to General Forcings

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The climate belongs to the class of non-equilibrium forced and dissipative systems, for which most results of quasi-equilibrium statistical mechanics, including the fluctuation-dissipation theorem, do not apply. In this paper we show for the first time how the Ruelle linear response theory, developed for studying rigorously the impact of perturbations on general observables of non-equilibrium statistical mechanical systems, can be applied with great success to analyze the climatic response to general forcings. The crucial value of the Ruelle theory lies in the fact that it allows to compute the response of the system in terms of expectation values of explicit and computable functions of the phase space averaged over the invariant measure of the unperturbed state. We choose as test bed a classical version of the Lorenz 96 model, which, in spite of its simplicity, has a well-recognized prototypical value as it is a spatially extended one-dimensional model and presents the basic ingredients, such as dissipation, advection and the presence of an external forcing, of the actual atmosphere. We recapitulate the main aspects of the general response theory and propose some new general results. We then analyze the frequency dependence of the response of both local and global observables to perturbations having localized as well as global spatial patterns. We derive analytically several properties of the corresponding susceptibilities, such as asymptotic behavior, validity of Kramers-Kronig relations, and sum rules, whose main ingredient is the causality principle. We show that all the coefficients of the leading asymptotic expansions as well as the integral constraints can be written as linear function of parameters that describe the unperturbed properties of the system, such as its average energy. Some newly obtained empirical closure equations for such parameters allow to define such properties as an explicit function of the unperturbed forcing parameter alone for a general class of chaotic Lorenz 96 models. We then verify the theoretical predictions from the outputs of the simulations up to a high degree of precision. The theory is used to explain differences in the response of local and global observables, to define the intensive properties of the system, which do not depend on the spatial resolution of the Lorenz 96 model, and to generalize the concept of climate sensitivity to all time scales. We also show how to reconstruct the linear Green function, which maps perturbations of general time patterns into changes in the expectation value of the considered observable for finite as well as infinite time. Finally, we propose a simple yet general methodology to study general Climate Change problems on virtually any time scale by resorting to only well selected simulations, and by taking full advantage of ensemble methods. The specific case of globally averaged surface temperature response to a general pattern of change of the CO<sub>2</sub> concentration is discussed. We believe that the proposed approach may constitute a mathematically rigorous and practically very effective way to approach the problem of climate sensitivity, climate prediction, and climate change from a radically new perspective.