



The Geometry of Chaotic Dynamics – A Complex Network Perspective

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Recently, several complex network approaches to time series analysis have been developed and applied to study a wide range of model systems as well as real-world data, e.g., geophysical or financial time series. Among these techniques, recurrence-based concepts and prominently ε -recurrence networks, most faithfully represent the geometrical fine structure of the attractors underlying chaotic (and less interestingly non-chaotic) time series. In this paper we demonstrate that the well known graph theoretical properties local clustering coefficient and global (network) transitivity can meaningfully be exploited to define two new local and two new global measures of dimension in phase space: local upper and lower clustering dimension as well as global upper and lower transitivity dimension. Rigorous analytical as well as numerical results for self-similar sets and simple chaotic model systems suggest that these measures are well-behaved in most non-pathological situations and that they can be estimated reasonably well using ε -recurrence networks constructed from relatively short time series. Moreover, we study the relationship between clustering and transitivity dimensions on the one hand, and traditional measures like pointwise dimension or local Lyapunov dimension on the other hand. We also provide further evidence that the local clustering coefficients, or equivalently the local clustering dimensions, are useful for identifying unstable periodic orbits and other dynamically invariant objects from time series. Our results demonstrate that ε -recurrence networks exhibit an important link between dynamical systems and graph theory.